Momentum Methods

15

The Eagle has landed.

— **Neil Armstrong,** *first landing on the moon, July 20, 1969.*

A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.

— **Max Planck,** *Nobel Prize Winner in Physics, developer of quantum theory, 1858–1947.*

15.1 General

A milestone in the development of modern science was the publication of Newton's three laws of motion. In brief, the three laws are as follows:

I. A body at rest tends to stay at rest and a body in motion tends to stay in motion unless acted upon by an external force.
II. When a force is applied to a free body, the rate at which the momentum changes is proportional to the amount of force applied. The direction in the change of momentum caused by the force is that of the line of action of the force.
III. For every action by a force, there is an equal but opposite reaction.

The above three principles form the basis for all nonrelativistic mechanics. Until vehicle velocities begin to reach about 3000 km/s, or about 1% the speed of light, the above three basic axioms of motion will do for every instance of vehicular accident analysis. Since the present relative velocities of impacting vehicles rarely exceed 200 km/hr or 0.0000185% the speed of light, there is little need of having to apply Lorentz transformations or other relativistic considerations to vehicular accident analysis.

Of course, this brings to mind the old joke about the pedantic motorist who told the judge that he did not run a red light; he was traveling fast enough that the red light appeared green to him. The judge, no stranger to

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Doppler effect calculations himself, simply agreed with the motorist and charged him $1 for every mile per hour over the speed limit.

In reviewing Newton's three laws, it is apparent that the analysis of momentum changes and forces is central to their application. In vehicular accidents, impact with a second vehicle usually causes changes in the direction and speed of both the involved vehicles. The analysis of the momentum and forces in such cases can provide valuable information about the accident, especially pre-impact speeds.

Sometimes when the available accident data are insufficient for other types of analyses to work, there is sufficient information to apply momentum methods to analyze the accident. Also, the application of momentum methods is an independent way of corroborating results from other types of analyses.

15.2 Basic Momentum Equations

The basic equation of engineering mechanics is the ever faithful:

$$F_A = m_A \, a_A \qquad \text{(i)}$$

where F_A = the applied force on item A, m_A = the mass of item A, and a_A = the resulting acceleration of item A.

Since the acceleration "a" is equal to the time derivative of the velocity, Equation (i) can be rewritten as follows to more closely parallel Newton's second law.

$$F_A = (d/dt)(mv)_A \qquad \text{(ii)}$$

where v = velocity.

The term "(mv)" is referred to as the momentum.

By rearranging the terms in Equation (ii), the following is obtained.

$$F_A(dt) = m_A(dv_A) \qquad \text{(iii)}$$

In the above equation, it is assumed that mass is constant during the time when the velocity changes. Thus, the mass derivative term, "v(dm)," is discarded.

The indefinite integration of both sides of Equation (iii), as done in Equation (iv), produces a term called the impulse. If a constant force is applied to a body for a certain time, then the impulse on that body is simply the force multiplied by the time.

The amount of impulse applied to a free body is equal to the change of momentum that results from its application.

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$$I_A = f[F_A(dt)] = m_A[v_{A2} - v_{A1}] \quad \text{(iv)}$$

Thus, a force applied on the body, "m_A," for a time duration "t" produces an impulse, "I_A." This impulse causes a change in the momentum of the body, which in this case equals the change in velocity vectors of the body.

Applying Newton's third law now allows a second body to be introduced into this analysis. If a second body is what is applying the force "F_A" to the first body, then the force on the second body will be equal but opposite in direction to that applied on the first body.

$$F_B = -F_A \quad \text{(v)}$$

where F_B = is the equal but opposite force applied on the second body.

Since the force applied on body A is equal and opposite to the force applied on body B, and since the forces on both bodies are applied for the same amount of time, then the impulse on body A is equal but opposite in direction to the impulse on body B.

$$-I_A = I_B \quad \text{(vi)}$$

$$-f\,[F_A(dt)] = f[F_B(dt)]$$

$$m_A[v_{A1} - v_{A2}] = m_B[v_{B2} - v_{B1}]$$

By rearranging the above terms, the equation for the conservation of linear momentum for a two-body impact is obtained.

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2} \quad \text{(vii)}$$

The left-hand side of Equation (vii) is the total linear momentum prior to impact, and the right-hand side is the total linear momentum after impact. Equation (vii) is the fundamental equation used in momentum analysis of vehicular accidents.

It should be noted that all the terms involving velocity, acceleration, etc. in the above equations are vectors and should be treated accordingly.

15.3 Properties of an Elastic Collision

Properly defined, a fully elastic collision between two bodies is one in which the deformation of each body obeys Hooke's law. Hooke's law states that within certain limits, the strain of a material is directly proportional to the applied stress causing the strain.

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