However, a more practical definition of an elastic collision between two bodies is: a collision in which it is observed that the two bodies return to the same shape they had prior to the collision; they do not significantly deform because of the collision.

The second definition is more practical than the first because it can be generally determined by visual inspection. If a vehicle has sustained little or no permanent deformation due to the collision, then the collision can be assumed to be elastic, barring any special impact energy absorption equipment that restores itself. Importantly, if the second definition is applied and the collision is deemed elastic, then the first definition can be applied to analyze the impact.

In an elastic collision between two bodies, there are four stages to the collision.

1. Contact is made between the two bodies.
2. Deformation occurs on both bodies according to Hooke's law reaching some maximum.
3. Recoil or restitution occurs as the stored "spring" energy in each body is released.
4. Separation between the two bodies occurs.

If each body in the collision reacts like a linearly elastic spring, then the following Hookian rule is applied:

$$F = -ks \qquad \text{(viii)}$$

where k = spring constant, s = amount of deformation of the body (a vector), and F = the applied force.

According to Newton's third law, the force of each body acting on the other is equal in magnitude, but opposite in direction. Thus,

$$-F_A = F_B \qquad \text{(ix)}$$

$$k_A s_A = -k_B s_B$$

The ratio of deformation between the two bodies is then as follows:

$$s_A/s_B = -k_B/k_A \qquad \text{(x)}$$

Since the time duration in which the two bodies are in contact with one another is the same, then the following also holds.

$$-I_A/I_B = \{m_A v_{A1} - m_A v_{A2}\}/\{m_B v_{B2} - m_B v_{B1}\} = k_A s_A/-k_B s_B \qquad \text{(xi)}$$

$$[m_A v_{A1} - m_A v_{A2}]/k_A = s_A \qquad \text{(xii)}$$

$$[m_B v_{B2} - m_B v_{B1}]/k_B = s_B$$

A review of Equations (x), (xi), and (xii) reveals two points. First, if the elastic constant "k" for both bodies is the same, then the resulting elastic deformation will be the same for both bodies. Secondly, the elastic deformation of each body is a function of its own change of momentum and elastic constant.

15.4 Coefficient of Restitution

During a two-body collision, there will be a point where the deformation of each body is at a maximum. When this point is reached, the two bodies will also have the same velocity. This common velocity is calculated by the following using the pre-impact momentum sum.

$$[m_A + m_B]u = m_A v_{A1} + m_B v_{B1} \qquad \text{(xiii)}$$

$$u = [m_A v_{A1} + m_B v_{B1}]/[m_A + m_B]$$

where u = the velocity at which both bodies are momentarily moving together.

This velocity can also be calculated from the momentum sum after the collision.

$$[m_A + m_B]u = m_A v_{A2} + m_B v_{B2} \qquad \text{(xiv)}$$

$$u = [m_A v_{A2} + m_B v_{B2}]/[m_A + m_B]$$

When the two bodies momentarily have the same velocity "u," the portion of the collision that occurs before it is considered the deformation portion of the collision, and the portion after it is considered the restitution portion. Therefore, the impulse for the deformation portion of the collision can be calculated using the definition of impulse given in Equation (iv).

$$I_{def} = m_A[v_{A1} - u] = m_B[u - v_{B1}] \qquad \text{(xv)}$$

where I_{def} = impulse during the deformation portion of the collision.

Likewise, the impulse for the restitution portion of the collision can be calculated from the definition of impulse given in Equation (iv).

$$I_{res} = m_A[u - v_{A2}] = m_B[v_{B2} - u] \quad \text{(xvi)}$$

The ratio of the restitution impulse to the deformation impulse is called the coefficient of restitution, "ε."

$$\varepsilon = I_{res/}I_{def} = [v_{B2} - v_{A2}]/[v_{A1} - v_{B1}] \quad \text{(xvii)}$$

It should be noted that the collisions being considered are assumed to be central collisions. If a collision is oblique, then the component vectors have to be separated. Those component vectors that are involved in the central collision obey the relations for the coefficient of restitution as noted above. Those components that are not involved in the central collision are handled separately. The latter vector components are often involved in causing rotation or doing frictional work such as sideswiping.

15.5 Properties of a Plastic Collision

In a nonelastic or plastic collision between two bodies, there are also four basic stages of the collision:

1. Contact is made between the two bodies.
2. Deformation occurs on both bodies reaching some maximum amount.
3. Some recoil or restitution may occur as any stored "spring" energy in either body is released.
4. Separation may occur between the two bodies if recoil is sufficient.

The main differences between the four stages of a plastic collision vs. an elastic one are as follows:

1. The deformation does not follow Hooke's law, that is, it does not elastically deform. It is likely that some elastic deformation will initially occur, but it may be insignificant as compared to the amount of plastic deformation that occurs.
2. There may not be any significant recoil or restitution.
3. The two bodies may not have sufficient recoil to separate; they may stay "stuck" to one another after impact and move as a unit.

A perfectly elastic collision has a coefficient of restitution of 1.0, and a perfectly plastic collision has a coefficient of restitution of zero. Between those two extremes are combinations of the two, where both some elastic and some