plastic deformation occurs. Obviously, the lower the magnitude of "ε," the less elastic deformation there is with respect to the plastic deformation.

Usually, any elastic deformation occurring in a collision takes place in the early part of contact between the two bodies. The plastic deformation then takes place after the elastic deformation range has been exhausted, and stops when the two bodies reach the "u" velocity [see Equation (xiii)]. Consequently, as the severity of the collision increases, the "ε" value approaches zero. Likewise, as the severity of the collision decreases, the "ε" value increases. Generally, the severity of the collision is proportional to the relative closing speed of the two vehicles.

The following coefficients of restitution, or "ε" values, have been found to occur in passenger vehicles either colliding with a fixed barrier or its equivalent. An example of a fixed barrier is a well-built brick wall that does not move or become damaged when it is impacted. Letting the subscripts "A" denote a vehicle and "B" denote a fixed barrier, then the coefficient of restitution of a fixed barrier is equal to the following:

$$\varepsilon = [v_{B2} - v_{A2}]/[v_{A1} - v_{B1}] = -v_{A2}/v_{A1},$$

because $v_{B2} = v_{B1} = 0$.

When forward or rearward velocities are 2.5 mph or less, "ε" may be considered to be 0.9 to 1.0. This is because U.S. federal law, as embodied in the National Traffic and Motor Vehicle Safety Act of 1966 with amendments, requires that bumpers on cars made after 1977 be able to withstand a 2.5 mph front or rear impact with a fixed barrier without significant damage occurring to the vehicle (49 CFR 581, 1987 edition).

For a time prior to 1978, vehicle bumpers were required by law to be able to withstand a 5 mph impact with a fixed barrier. However, this was rescinded in 1977, and the current, lower standard was then established. Prior to 1966, there were no legally mandated bumper standards. Thus, no similar "ε" value can be categorically assigned to vehicle bumpers made prior to 1966.

From experience, it has been found that at impact speeds of about 25 mph, an "ε" value of 0.2 is typical for most passenger cars. At impact speeds of about 35 mph, an "ε" value of 0.1 is typical, and at speeds exceeding 50 mph, an "ε" value of about 0.004 is typical.

Interestingly, when the above experience-derived values of "ε" are plotted against the vehicle impact speed, they tend to closely follow the following equation:

$$\varepsilon = e^{-0.065s} \qquad \text{(xviii)}$$

where ε = the coefficient of restitution, s = the vehicle's speed in mph, and e = logarithmic "e," i.e., e = 2.718.

Plate 15.1 Side impact. Note bumper imprint in crush damage along door.

15.6 Analysis of Forces during a Fixed Barrier Impact

In experimental tests, it has been found that when a 1980 Chevy Citation weighing 3130 lb was driven into a fixed barrier at 48 mph, the front end crushed 40.4 in deep across the front of the car.

Using Equation (xviii), it is found that this impact velocity would have a coefficient of restitution of about 0.044. Using this "e" value, the calculated rebound velocity would have been about 2 mph. For all practical purposes, this impact would be deemed a plastic collision.

The pre-impact momentum of the car would have been:

$$[(3130 \text{ lbf})/(32.17 \text{ ft/sec}^2)][(70.4 \text{ ft/sec})] = 6850 \text{ lbf-sec.}$$

The postimpact momentum of the car would have been:

$$[3130 \text{ lbf})/(32.17 \text{ ft/sec}^2)][3 \text{ ft/sec}] = 292 \text{ lbf-sec.}$$

The net change in momentum due to the impact was then 6558 lbf-sec.

After contact with the wall, the car came to a stop in 40.4 inches. Assuming that the velocity of the vehicle decreases linearity (i.e., constant deceleration), then the equation for the speed of the car during impact would be as follows:

$$70.4 \text{ ft/sec} - (1.743 \text{ ft/in-sec})x = V$$

where x = the amount of crush that has occurred, and V = the velocity at that point during the impact.

If the average velocity during the impact is simply "(1/2)(70.4 ft/sec)," then the time required to stop in 40.4 inches would be 0.0956 seconds. Thus, the actual impact is literally over in less than the blink of an eye.

Assuming that the force applied by the wall to stop the car was constant, then the impulse during the impact would be:

$$F(0.0956 \text{ sec}) = 6558 \text{ lbf-sec.}$$

$$F = 68{,}598 \text{ lbf.}$$

If the force would have been applied more or less evenly across the front of the car, and the front is assumed to be about 72 inches wide and about 30 inches in height, then an equivalent average pressure on the wall to resist the car impact would have been 32 psi.

The deceleration rate during the impact is estimated as follows:

$$d = (1/2)at^2$$

$$d = 40.4 \text{ in} \quad t = 0.0956 \text{ sec}$$

$$a = 8841 \text{ in/sec}^2 = 737 \text{ ft/sec}^2 = 22.3 \text{ g's}$$

From this simple analysis, a few items are noted. First, if the crush depth is increased, then the deceleration can be decreased and the time of impact increased. For example, if the crush depth had been 50 in instead of the 40.4 in, then the time of impact would have been 0.118 sec, and the deceleration would have been 18.6 g's instead of 22.3 g's. The 9.6 in increase in crush depth, a 24% increase, would have reduced the deceleration by 17%. This is significant. The degree of injury sustained in a vehicle impact by the occupants is a direct function of the impact deceleration.

15.7 Energy Losses and "ε"

When two bodies collide, some of the total kinetic energy is dissipated in plastic deformation. The amount of energy loss is given by the following.

$$(1/2)(m_A v_{A1}^2) + (1/2)(m_B v_{B1}^2) = E_{Loss} + (1/2)(m_A v_{A2}^2) + (1/2)(m_B v_{B2}^2) \quad \text{(xix)}$$

$$E_{Loss} = (1/2)m_A(v_{A1}^2 - v_{A2}^2) + (1/2)m_B(v_{B1}^2 - v_{B2}^2)$$

In the above equation, the "v^2" terms are considered either vector dot products, or simply the absolute values of the velocity vectors squared.

The coefficient of restitution, "ε," is given by the following:

$$\varepsilon = I_{res}/I_{def} = [v_{B2} - v_{A2}]/[v_{A1} - v_{B1}].$$

Rearranging the above relation for "ε" to favor postimpact velocity terms gives the following:

$$v_{B2} = \varepsilon[v_{A1} - v_{B1}] + v_{A2} \quad \text{and} \qquad \text{(xx)}$$

$$v_{A2} = v_{B2} - \varepsilon[v_{A1} - v_{B1}].$$

Using the above value for "v_{A2}," the following is generated.

$$(v_{A1}^2 - v_{A2}^2) = v_{A1}^2 - [v_{B2}^2 - 2(\varepsilon)(v_{A1} - v_{B1})(v_{B2}) + \varepsilon^2(v_{A1} - v_{B1})^2] \qquad \text{(xxi)}$$
$$(v_{A1}^2 - v_{A2}^2) = v_{A1}^2(1 - \varepsilon^2) - v_{B2}^2 + v_{A1}v_{B2}(2\varepsilon) - v_{B1}v_{B2}(2\varepsilon) + v_{A1}v_{B1}(2\varepsilon^2) - v_{B1}^2(\varepsilon^2)$$

Similarly, using the value in Equation (xx) for "v_{B2}," the following is generated.

$$(v_{B1}^2 - v_{B2}^2) = v_{B1}^2(1 - \varepsilon^2) - v_{A2}^2 + v_{B1}v_{A2}(2\varepsilon) - v_{A1}v_{A2}(2\varepsilon) + v_{A1}v_{B1}(2\varepsilon^2) - v_{A1}^2(\varepsilon^2) \qquad \text{(xxii)}$$

Substituting the above velocity difference terms, Equations (xxi) and (xxii), into Equation (xix) gives the following for the energy loss

$$E_{Loss} = (1/2)(m_A)[v_{A1}^2(1 - \varepsilon^2) - v_{B2}^2 + v_{A1}v_{B2}(2\varepsilon) - v_{B1}v_{B2}(2\varepsilon) + v_{A1}v_{B1}(2\varepsilon^2) - v_{B1}^2(\varepsilon^2)] + (1/2)(m_B)[v_{B1}^2(1 - \varepsilon^2) - v_{A2}^2 + v_{B1}v_{A2}(2\varepsilon) - v_{A1}v_{A2}(2\varepsilon) + v_{A1}v_{B1}(2\varepsilon^2) - v_{A1}^2(\varepsilon^2)] \qquad \text{(xxiii)}$$

Rearranging Equation (xxiii) to sort out the "ε" terms, and substituting to favor postimpact velocities, gives the following expression:

$$E_{Loss} = [(1/\varepsilon^2) - 1][(m_A m_B)/2(m_A + m_B)][v_{B2} - v_{A2}]^2 \qquad \text{(xxiv)}$$