Similarly, if pre-impact terms are favored, the following expression can be derived:

$$E_{Loss} = [(1 - \varepsilon^2)/2][(m_A m_B)/(m_A + m_B)][v_{A1} - v_{B1}]^2 \qquad \text{(xxv)}$$

When $\varepsilon = 1$, both Equations (xxiv) and (xxv) become zero, indicating again that a fully elastic collision is one in which there is no energy loss. When $\varepsilon = 0$, Equation (xxx) is indeterminate due to division by zero. However, Equation (xxv) is solvable.

To be a half-plastic and half-elastic collision, "ε" must equal 0.707. In a passenger car colliding with a fixed barrier, such a half-plastic and half-elastic collision would occur at a speed of about 5.33 mph. A collision with only 10% elastic energy would have an "ε" of 0.316 and would occur at a speed of about 17.7 mph. It is readily apparent, then, that in most vehicular collisions, the predominant deformation mode is plastic. It is also apparent that in most vehicular collisions, recoil is a small effect.

15.8 Center of Gravity

In all of the foregoing analyses, it has been tacitly assumed that the masses of the bodies were point masses. They were considered to be like minute billiard balls, bouncing off one another. When such equations are applied to vehicles involved in collisions, they then apply to the centers of mass of the vehicles.

Thus, all of the previous equations were based on the premise of straight central impact, where the line of action between the bodies was through the center of gravity of each body.

The distribution of load on a vehicle is not necessarily such that the center of mass is at the geometric center of the vehicle. If "Q" is the load carried by the rear axle and "R" the load carried by front axle, then the center of gravity (C.G.) in a two-axle vehicle will be found as follows.

$$a = Qb/(Q + R) = Qb/W = b/(1 + R/Q) \qquad \text{(xxvi)}$$

where a = distance from the front axle to the center of gravity of the vehicle, b = distance between axles, and W = total weight of the vehicle.

For example, in a car with a 106-in wheel base and 60% of the curb weight carried by the front axle, the C.G. is located 42.4 in away from the front axle in a direction toward the rear of the car. Alternately, the C.G. is located 13.6 in away from the midpoint of the wheelbase toward the front of the car.

The above, of course, assumes that the C.G. is found on a line that bisects the length of the vehicle. For most purposes, this will be sufficient. If, however, there is some type of asymmetrical load on the vehicle, perhaps a load shift during the collision, it might be necessary to determine the C.G. using the point mass method. In this method, the weight carried under each tire is determined, and the coordinates of the center of gravity are determined by the following:

$$x_c = [\Sigma m_i x_i]/[\Sigma m_i] \quad y_c = [\Sigma m_i y_i]/[\Sigma m_i] \tag{xxvii}$$

where x_c and y_c are the coordinates of the C.G.

Typically, the point of origin from which the "x" and "y" distances of the mass points are measured is one of the tire contact points. This causes the "x" and "y" values for that tire to be zero, thus simplifying some of the calculations.

Sometimes it is important also to know the position of the C.G. with respect to the "z" axis, that is, the vertical axis. However, while the center of gravity with respect to the horizontal plane and "x and y" coordinates is relatively easy to locate, the "z" coordinate requires more effort.

Perhaps the simplest, although not the easiest method of determining the C.G. with respect to all three coordinates, utilizes a crane. The vehicle is lifted with a three-point or four-point harness and is suspended by a single cable attached to the harness. Preferably, the vehicle should be somewhat tilted or otherwise not parallel with the ground. While suspended, a photograph of the vehicle is taken from a set position. A second lift is then similarly done except the vehicle is tilted differently from the first lift. With the vehicle facing the same side to the camera, a second photograph is taken.

The single cable that holds up the vehicle acts as a plumb line in the photographs. The line of action of the cable runs directly through the C.G. of the vehicle. Thus, the line of this cable should be graphically extended so that it runs down through the image of the vehicle. When the two photographs are superimposed, the point where the two plumb lines intersect on the vehicle is the three-coordinate C.G. If the photographs were taken with the same side facing the camera, it is an easy matter to locate this point on the photographs and then determine where it is on the car itself.

Unfortunately, an apparatus for lifting cars and taking their pictures is not always handy, or cheap. Thus, an alternate method may need to be employed that has a lessor equipment requirement.

Knowing that the "z" component of the C.G. will be directly above the "x and y" components, then the "z" component can be found by jacking up the vehicle with a scale under the jack to read the load. The calculations are easiest when the jack is placed in line with either the longitudinal or trans-