verse axes of the C.G. By jacking up the vehicle until the initial load on the jack significantly reduces, and noting the angle of the vehicle, the "z" component can be found by simple statics.

If a person continued to jack the car up until it is balanced on the two wheels making contact, the C.G. would be directly over the axis of the two wheels, and the load on the jack would be zero.

By taking a photograph with the view parallel to the axis of the contacting wheels, and drawing a plumb line on the photograph that originates at the axis of the contacting wheels, the line would pass through the C.G. Assuming that the "z" component lines directly above the "x and y" C.G. coordinates, the intersection of the drawn plumb line with the line that bisects the vehicle is the C.G. The "z" component then can be scaled from the photograph.

As a rule of thumb, most late model passenger cars have a "z" coordinate located about 2 feet from the ground. Regular sized pickup trucks, e.g., Ford 100, have a "z" coordinate located about 2.5 feet above the ground.

15.9 Moment of Inertia

The moment of inertia of a vehicle can be determined experimentally by suspending the vehicle from a single cable and variously spinning it and measuring the rate of rotation. This experimental method takes advantage of the relation:

$$F \times r = I\alpha \qquad \text{(xxviii)}$$

where F = the applied force, r = distance from the C.G., I = moment of inertia, and α = angular acceleration.

In this case, the cross product between two vectors is used, which is denoted by the "×."

In general a constant force is applied at one of the far ends of the vehicle, perpendicular to the lever arm between the point of application and C.G. of the vehicle. The final angular velocity, "ω_f," is measured and its value is divided by the time the force was applied to obtain "α," the angular acceleration. Thus, the moment of inertia is found by the following:

$$I = [F \times r]/[\ \omega_{f/}t]. \qquad \text{(xxix)}$$

In Section 15.8 the method for finding the center of gravity by suspending the vehicle from a cable was discussed. Since the additional effort necessary to find "I" is rather small in comparison to the trouble of the setup, it is recommended that they both be done at the same time. However, if a person is unable to undertake hoisting a car at the end of a cable and spinning it, there are several methods that can be used to provide reasonable estimates of "I."

The first method assumes that the distribution of mass in the vehicle is more or less even over the whole projected x-y area and that the center of gravity is near the geometric center.

$$m_{areal} = m/A \qquad \text{(xxx)}$$

where A = area of the vehicle projected onto the horizontal plane, m = mass of the vehicle, and m_{areal} = mass per unit area.

For a rectangular object the moment of inertia is:

$$I_{xx} = m_{areal}bh^{3/}12 = m_{areal}Ah^{2/}12 = mh^{2/}12 \qquad \text{(xxxi)}$$

where b = width, h = length, and I_{xx}= moment of inertia about x-x axis.

Using the above method, for a 1988 Buick Century, which is 189 in long and 69 in wide, with a curb weight of 2950 lbs, its moment of inertia would be as follows:

$$m = 2950 \text{ lbf}/(386 \text{ in/sec}^2) = 7.642 \text{ lb-sec}^2\text{/in}$$

$$I_{xx} = (7.642 \text{ lb-sec}^2\text{/in})(189 \text{ in})^2/12 = 22{,}750 \text{ lb-sec}^2\text{-in.}$$

A second method considers the loads carried by the axles as point masses located half the wheelbase from geometric center.

$$I_{xx} = (x)m(l/2)^2 + (1\text{-}x)m(l/2)^2 = ml^2/4 \qquad \text{(xxxii)}$$

where l = length of wheel base, x = fraction of mass carried by front axle, and (1-x) = fraction of mass carried by rear axle.

Using the same 1988 Buick Century, whose wheelbase is 105 in, the "I" value is calculated as follows.

$$I_{xx} = (0.65)(7.642 \text{ lb-sec}^2\text{/in})(105 \text{ in}/2)^2 + (0.35)(\text{lb-sec}^2\text{-in})(105 \text{ in}/2)^2$$

$$I_{xx} = 21{,}063 \text{ lb-sec}^2\text{-in.}$$

A third method assumes that the engine is primarily responsible for the uneven distribution of load between axles. Thus, the mass of the engine is