deducted from the total vehicle mass and an “I” value is computed for the “engine-less” vehicle. The engine mass is then added in as a point mass.

$$I_{xx} = 2(1\text{-}x)mh^2/12 + (2x\text{-}1)m(l/2)^2 \quad \text{(xxxiii)}$$

where x = fraction of load on front axle, h = overall length of vehicle, l = wheel base length, and m = mass of vehicle.

Again, using the 1988 Buick Century, the following “I” value is computed.

$$I_{xx} = 2(0.35)(7.642 \text{ lb-sec}^2\text{-in})(189 \text{ in})^2/12 + (0.30)(7.642 \text{ lb-sec}^2\text{-in})(105 \text{ in}/2)^2$$

$$I_{xx} = 15{,}923 \text{ lb-sec}^2\text{-in} + 6{,}319 \text{ lb-sec}^2\text{-in} = 22{,}242 \text{ lb-sec}^2\text{-in.}$$

When a vehicle is carrying an unusually heavy load in the trunk or elsewhere, the additional mass can be figured in the computation for “I” in the manner used for the engine mass.

Of the three methods, method three seems to work best for passenger cars, where the mass distribution is fairly even except for the engine. The first method seems to work best for truck trailers and vehicles where the general mass distribution is relatively even and there are no large concentrations of mass. Method two works well when there are heavy loads over the axles, like in a loaded pickup truck (engine over front axle, load over rear). However, as demonstrated by the 1988 Buick Century calculations, all three methods give similar results.

One additional item to note is that the “I” values discussed above assumed that the x-x axis was in the geometric center of the vehicle. As such, it was assumed that rotation would be about the center of the vehicle. If, however, rotation occurs where the center of rotation is about the rear axle or the front axle, then the parallel axis theorem will have to be applied to properly adjust the “I” to the point of rotation. The parallel axis theorem is given below.

$$I_{ii} = I_{xx} + mr^2 \quad \text{(xxxiv)}$$

where I_{ii} = moment of inertia about axis i-i, r = distance between axis i-i and axis x-x, and m = mass of the vehicle.

15.10 Torque

Again, starting with the basic equation of engineering:

$$F = ma.$$

Let us now apply the force not to an idealized ball or point of mass, but to a stiff object with some length. Further, let the force be applied at a point so that the line of action of the force does not pass through the center of mass. In that case, then the following occurs.

$$T = F \times r = (ma) \times r = m(a \times r) \tag{xxxv}$$

where T = torque, r = vector distance from center of mass, F = force applied at "r," m = total mass of item, and a = acceleration in the direction of the force.

As before, the "×" denotes a cross-product vector multiplication.

The rotation of a body around its center of mass can be constant or variable. When the rotation is constant, the rotation rate is usually given in radians per second. When the rotation rate varies, the rate of change is quantified by the angular acceleration, which is given in radians per second. The relationship between angular velocity, "w," and angular acceleration, "α," is given as follows.

$$\alpha = \Delta\omega/\Delta t \tag{xxxvi}$$

where Δ = an incremental change and t = time.

The tangential velocity at distance "r" from the center of mass is given by the following:

$$v_T = \omega \times r. \tag{xxxvii}$$

Similarly, the tangential acceleration at distance "r" from the center of mass is given by the following:

$$\alpha_T = (d/dt)(\omega \times r) = \alpha \times r. \tag{xxxviii}$$

Substituting that back into Equation (xxxv) gives the following:

$$T = F \times r = m(\alpha \times r) \times r \tag{xxxix}$$

Collecting terms and simplifying gives:

$$T = F \times r = (mr^2)\alpha = I_{xx}\alpha \tag{xl}$$

where I_{xx} = the moment of inertia of a point mass about the axis of rotation.