If $\varepsilon = 0$, a fully plastic collision, it is seen that Equations (xliv) and (xlv) are undefined due to the division by zero. However, when $\varepsilon = 1$, it is automatically known that $v_{A2} = v_{B2} = u$.

In a similar fashion, if Equation (xvii), the relation for the coefficient of friction, is alternately solved for "v_{A2}" and "v_{B2}," and the results are substituted back into the equation for the conservation of momentum, Equation (vii), then the following is obtained:

$$v_{A2} = [(m_A - \varepsilon m_B)/(m_A + m_B)]v_{A1} + [(m_B + \varepsilon m_B)/(m_A + m_B)]v_{B1} \quad \text{(li)}$$

$$v_{B2} = [(m_A + \varepsilon m_A)/(m_A + m_B)]v_{A1} + [(m_B - \varepsilon m_A)/(m_A + m_B)]v_{B1} \quad \text{(lii)}$$

Equations (li) and (lii) indicate that if the velocities just prior to collision are known, and if the "ε" value for the collision is known, then the individual velocities after impact can be calculated. This is also a very useful analytical tool.

The velocities of the two vehicles prior to impact, as reported by the drivers or witnesses, can be substituted into "v_{A1}" and "V_{B1}." Then, the after-impact velocities can be solved. The calculated after-impact velocities can then be compared to the actual accident scene information. If they match reasonably well, the drivers' accounts are validated. If they do not match, then one or both of the claimed pre-impact velocities are incorrect.

As before, these equations can also be arranged in the form:

$$v_{A2} = [w]v_{A1} + [x]v_{B1} \quad \text{(liii)}$$

$$v_{B2} = [y]v_{A1} + [z]v_{B1}.$$

As before, it is found that the following relationship holds for the scalars:

$$w + x = y + z = 1. \quad \text{(liv)}$$

In a fully elastic collision, Equations (li) and (lii) simplify to the following:

$$v_{A2} = [(m_A - m_B)/(m_A + m_B)]v_{A1} + [(2m_B)/(m_A + m_B)]v_{B1} \quad \text{(lv)}$$

$$v_{B2} = [(2m_A)/(m_A + m_B)]v_{A1} + [(m_B - m_A)/(m_A + m_B)]v_{B1} \quad \text{(lvi)}$$

In a fully elastic collision, it is seen that in addition to Equation (liv), the additional relations hold between the scalars:

$$w = -z \tag{lvii}$$

$$x + y = 2$$

$$y - w = 1$$

$$x - z = 1.$$

The reader will undoubtedly note the similarity of the relations in Equation (lvii) with those of Equation (l).

Similarly, when the collision is fully plastic and $\varepsilon = 0$, the following holds:

$$v_{A2} = v_{B2} = [(m_A)/(m_A + m_B)]v_{A1} + [(m_B)/(m_A + m_B)]v_{B1} \tag{lviii}$$

15.13 Estimation of a Collision Coefficient of Restitution from Fixed Barrier Data

When a vehicle impacts a fixed barrier, it is presumed that the fixed barrier does not crush or otherwise absorb any significant amount of kinetic energy from the vehicle. The fixed barrier pushes back against the vehicle during impact with a force equal to that being applied by the vehicle. In essence, since no work can be done on the fixed barrier, the vehicle does work on itself.

Earlier, Equation (xxv) was derived to show the relationship between energy dissipated by the collision and the coefficient of restitution. For convenience, the equation is shown again below.

$$E_{Loss} = [(1 - \varepsilon^2)/2][(m_A m_B)/(m_A + m_B)][v_{A1} - v_{B1}]^2 \tag{xxv}$$

If it is assumed that a fixed barrier essentially has an "infinite" or immovable mass, and the wall has no initial speed prior to the collision, then the equation reduces to the following:

$$E_{Loss} = [(1 - \varepsilon^2)/2][m_A][v_{A1}^2]. \tag{lix}$$

When $\varepsilon = 0$, a fully plastic collision, all of the initial kinetic energy is dissipated as work in crushing the vehicle. When $\varepsilon = 1.0$, none of the initial kinetic energy is dissipated as work; all of it goes into the rebound of the vehicle.

While not exactly contributing to the point at hand, the following aside may be interesting to the reader. By substituting Equation (xviii) into (lix), a function for collision energy loss against a fixed-barrier collision can be derived that is solely dependent upon initial speed: