$$E_{Loss} = [(1 - e^{-0.130s})/2][m_A][s_{A1}^2] \qquad \text{(lx)}$$

where s = velocity of Vehicle "A" given in mph and e = 2.718 etc.

An alternate version of the above equation where velocity is given in ft/sec is as follows:

$$E_{Loss} = [(1 - e^{-0.089v})/2][m_A][v_{A1}^2]. \qquad \text{(lxi)}$$

However, getting back to the point of this section, if it is assumed that crush caused by collision with the fixed barrier is proportional to the change in kinetic energy of the vehicle, then:

$$s_A \cdot F = (1/2)m_A(\Delta v_A^2) = U = E_{Loss} \qquad \text{(lxii)}$$

where U = work done in crushing the vehicle, $s_A$ = amount of crush in the vehicle, F = the average force applied to the vehicle during collision with the fixed barrier, and $\Delta v_A^2 = v_{A2}^2 - v_{A1}^2$.

When two vehicles collide head on, the total work done in crushing is as follows:

$$U = (1/2)m_A(\Delta v_A^2) + (1/2)m_B(\Delta v_B^2) = s_A \cdot F + s_B \cdot F. \qquad \text{(lxiii)}$$

The force "F" applied between the two vehicles during collision is the same in magnitude but opposite in sign.

Now, if in the above equation, Vehicle "B" is sitting still with respect to Vehicle "A" (i.e., $\Delta v_B^2 = 0$), then the crush damage on both vehicles would be due to the action of Vehicle "A" alone. As can be seen in comparing Equations (lxii) and (lxiii), this means that the crush on Vehicle "A" in colliding with Vehicle "B" would be about half that of Vehicle "A" colliding with a fixed barrier at the same speed.

$$1/2 = v_{Aeq}^{2/}v_{A1}^2 \quad \text{or} \quad v_{Aeq} = (0.707)v_{A1} \qquad \text{(lxiv)}$$

where $v_{Aeq}$ = impact velocity of "A" with a fixed barrier and $v_{A1}$ = impact velocity of "A" with Vehicle "B," which is sitting still.

Thus, the coefficient of restitution for the above collision situation can be estimated by converting "$(0.707)v_{A1}$" into mph units, and substituting into Equation (xviii).

In a similar fashion, equivalent fixed-barrier impact velocities can be derived for other situations. For example, if the two vehicles have about the same mass and collide with the same velocity head on, then the equivalent fixed-barrier impact velocity is equal to the actual impact velocity.

## 15.14 Discussion of Coefficient of Restitution Methods

In general, the pre-impact velocities in a two-vehicle collision can be solved by proceeding as follows:

1. A reasonable "guess" as to the velocities of the vehicles is made. It is important to note that the "guess" is for the central impact velocities. If the collision was oblique, then it may be necessary to separate the central velocity vector components and handle the others separately.
2. A tentative value for "ε" is calculated from the first guess.
3. Using the tentative value for "ε," Equations (xliv) and (xlv) are solved.
4. A new value of "ε" is calculated based on the outcome of Equations (xliv) and (xlv).
5. If the new value of "ε" matches the tentative value of "e" first assumed, and the calculated velocities reasonably match the "guessed" values, then a reasonable solution has been obtained.
6. If the variance is too great, then a new "guess" of the velocities is made and the process is repeated until a reasonable convergence occurs.

However, in solving pre-impact velocities using the above method, Equations (xliv) and (xlv) become extremely sensitive to small variations in the value of "ε" when "ε" is very small. Of course, when "e" is small, the collision is mostly plastic and has occurred at a relatively high speed.

Thus, when "ε" is very small and subject to imprecision, an alternate approach may be advisable.

1. Using Equations (li) and (lii), the pre-impact velocities are assumed, or taken to be the values as stated by the drivers or witnesses.
2. An "ε" value is determined from equivalent barrier impact velocity computations, or simply assumed.
3. The postimpact velocities are calculated.
4. From the actual police report data, the postimpact velocities are determined. These velocity vectors are then compared to the calculated velocity vectors. If they match reasonably, a satisfactory solution has been found.
5. If the postimpact velocities do not match reasonably, new preimpact velocity vectors are assumed and the process is repeated until convergence is achieved.

## Further Information and References

*Automobile Side Impact Collisions – Series II* by Severy, Mathewson and Siegel, University of California at Los Angeles, SAE SP-232. For more detailed information please see Further Information and References in the back of the book.

*Mechanical Design and Systems Handbook*, Harold Rothbart, Ed., Chapter 16, McGraw-Hill, New York, 1964. For more detailed information please see Further Information and References in the back of the book.

*Motor Vehicle Accident Reconstruction and Cause Analysis*, by Rudolf Limpert, 2nd ed., The Michie Company, Charlottesville, Virginia, 1984. For more detailed information please see Further Information and References in the back of the book.

*Symposium on Vehicle Crashworthiness Including Impact Biomechanics*, Tong, Ni, and Lantz, Eds., American Society of Mechanical Engineers, AMD-Vol. 79, BED-Vol. 1, 1986. For more detailed information please see Further Information and References in the back of the book.

*The Traffic Accident Investigation Manual*, by Baker and Fricke, 9th ed., Northwestern University Traffic Institute, 1986. For more detailed information please see Further Information and References in the back of the book.