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Mathematical Study of Heat Transport in Human Dermal Regions Under Normal and Tumour Conditions

Aijaz A. Najar*[1], Rouf Gulzar[2], and Inderdeep Singh[2]

[1]Department of Matematics GDC (Boys) Baramulla, J & K, India. aijaznajar25@gmail.com*

[2]Department of Mathematics, Sant Baba Bagh Singh University, Jalandhar, 144030, Punjab, India.
abdulrouf2005@gmail.com, inderdeeps.ma.12@gmail.com

***Abstract*—The heat transfer models have nowadays engulfed a large domain of scientific research and many researchers are actively involved in it. The understanding of effect of temperature variation and thermal properties of skin under different conditions has a great role in day to day life. The present paper studies the temperature distribution in human dermal regions with a temperature dependent perfusion and an oscillatory boundary condition. A finite difference technique with suitable boundary and interface conditions is used for predicting the temperature pattern inside the normal and tumour skin tissue layers. It has been observed that the presence of sinusoidal heat flux diminishes the temperature amplitude along the tissue depth. The increase in perfusion rate increases the rate of heat loss from the tissues to the blood.**

***Index Terms*—Tumour tissue, Mathematical model, Diffusion, FDM, Tissue Temperature.**

I. Introduction

The heat distribution of skin has been extensively studied for various diagnosis in medical sciences (Lang et al. (1999), Park et al. (2007), Shih et al. (2007)), or even for the study of the physiological functions of healthy individuals (Shusterman et al. (1997)). It has nowadays engulfed a large domain of scientific research and many researchers are actively involved in it. There is a long standing interest in thermal properties of the skin (Stoll, A. M.) in order to understand conditions leading to thermal damage to skin, usually involving contact with the hot objects (Stoll et al. (1979)). Investigations of such bioheat transfer problems requires the elevation of temporal and spatial distributions of temperature. Moreover during the hyperthermia the transfer of heat may be affected by the flow of blood and the vascular geometry. Investigation of thermal properties of skin (Kengne et al. (2012), Dai et al. (2006), El-dabe et al. (2003), Gowrishankar et al. (2004)) leading to thermal injuries are usually studied through the classical equation of Pennes' bioheat equation (Pennes (H. H.)). The circulating blood is about 10% of the total blood volume of a normal human being, therefore the convection diffusion and perfusion of blood plays an important role in the bioheat process (Hall & Guyton (2012)). Due to the seminal work of Pennes' in 1948 (Pennes, H. H.), the researchers like (Dai et al. (2006), El-dabe et al. (2003), Frahm et al. (2010)) began to study the heat transfer in many biological systems with the help of this fundamental equation called Pennes' bioheat equation(Pennes, H. H.)

$$\rho c\frac{\partial T}{\partial t} = div(kgradT) - c_b w(T - T_b) + Q_m \qquad (1)$$

where ρ, c, k are the density, specific heat and thermal conductivity of tissue respectively, T_b is arterial blood temperature, t is time, c_b is the specific heat of blood, w is the perfusion rate per unit volume of the blood, Q_m is the metabolic heat generation per unit volume.

The most recent work using an explicit form of finite difference method for estimating the temperature variation in human dermal regions has been studied by Khanday and Fida (Khanday & Fida (2015)) and the temperature variation in the human body along with tumour conditions have been studied recently (Babita & Neeru (2018), Kamangar et al. (2019),Khalid et al. (2017)). However, due to the complex vascular structure in particular and whole body in general,always paves a way for the researchers to improve the existing theoretical models for better understanding and for accurate clinical results. The present paper deals with the temperature distribution in human dermal regions with a temperature dependent perfusion and an oscillatory boundary condition. A finite difference scheme with suitable boundary and interface conditions is used for predicting the temperature pattern inside the normal and tumor skin tissue layers.

*Corresponding Author

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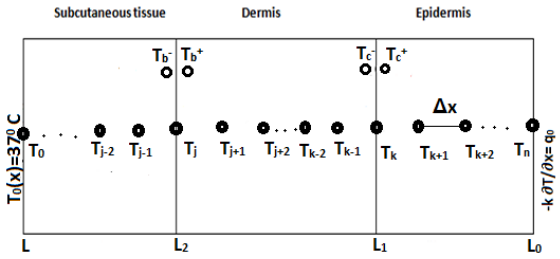

Fig. 1. Schematic diagram showing the discretization of the domain of human dermal regions.

II. Mathematical Model

To study the heat distribution in human dermal regions through three layers as shown in Fig. (1), we consider size of the three layers as

- Subcutaneous tissue $(L \leq x \leq L_2)$ of length l_1
- Dermis $(L_2 \leq x \leq L_1)$ of length l_2
- Epidermis $(L_1 \leq x \leq L_0)$ of length l_3

We consider a special case of the one-dimensional (1-D) Pennes' bioheat transfer equation with a constant thermal conductivity of the tissue and temperature dependent perfusion as

$$k\frac{\partial^2 T}{\partial x^2} = \rho c\frac{\partial T}{\partial t} + c_b w_b(T - T_a) + \rho_b c_b w_m(T)(T - T_a) + Q_m + Q_h \tag{2}$$

Here, $x(0 \leq x \leq L)$ gives the distance from the skin surface to the body core (in m), t is the time (in s), and $T = T(x,t)$ measures the local temperature at depth x from the surface at time t; L is the distance (in m) between the skin surface and the body core. Therefore, we assume in our investigation that the skin surface is defined at $x = L_0$ while the body core at $x = L$. Q_h is the heat source at the skin surface.

The 1-D case of Pennes' bioheat transfer equation is a good approximation when heat mainly propagates in the direction perpendicular to the skin surface. Comparing Eq. (1) and Eq. (2), it is clear that temperature-dependent blood perfusion reads

$$w = w_b + \rho_b w_m(T) \tag{3}$$

To completely determine the temperature distribution, it is necessary to associate boundary conditions with the the partial differential Eq. (2). In our case, we associate with Eq. (2) the oscillatory heat flux boundary condition which is described as follows Shih et al. (2007)

$$-k\frac{\partial T}{\partial x} = q_0 e^{\omega t} + \rho_b w_m(T) \tag{4}$$

where q_0 and ω are the heat flux on the skin surface and the heating frequency respectively and $q_0 e^{it}$ is the time-dependent surface heat flux. No heat loss is assumed at $x = L$ and the body core temperature is regarded as constant (T_c) by considering the fact that the biological body tends to keep its core temperature stable for normal function and therefore, we can take

$$T(x,t)|_{x=L} = T_c. \tag{5}$$

TABLE I
Properties of various types of tissues Kengne et al. (2012)

Tissue	**Thermal Conductivity**(k) $[W/m/^0C]$	**Density** (ρ) $[Kg/m^3]$	**Specific Heat**(c) $[Ws/Kg/^0C]$
Muscle	0.642	1000	$3,500$
Tumor	0.642	1000	$3,500$
Dermis	0.450	1200	$3,300$
Subcutaneous	0.190	1000	$2,675$

TABLE II
Parameters and their physiological values used in the study Kengne et al. (2012),Gowrishankar et al. (2004)

Parameter	**Unit**	**Value**
Heat flux (q_0)	W/m^2	500
Metabolic heat generation (Q_m)	W/m^3	33800
Density of blood(ρ_b)	Kg/m^3	1060
Specific heat of blood(c_b)	$Ws/Kg/^0C$	3500
Width of skin(L)	m	0.02
Arterial blood temperature(T_b)	^0C	37^0C
Thickness of subcutaneous tissue(l_1)	μm	1800
Thickness of Dermis(l_2)	μm	2000
Thickness of epidermis(l_3)	μm	80
Thermal conductivity of subcutaneous tissue(k_1)	$Wm^{-1}\ ^0C^{-1}$	0.19
Thermal conductivity of dermis(k_2)	$Wm^{-1}\ ^0C^{-1}$	0.45
Thermal conductivity of epidermis(k_3)	$Wm^{-1}\ ^0C^{-1}$	0.23

III. Solution

Since the blood perfusion is temperature dependent, the model Eq. (2) is non-linear. Thus the analytical solution is not applicable. Therefore, we employ a finite difference technique for obtaining the solution of Eq. (2) along with conditions given by Eq. (3), Eq. (4), & Eq. (5). We take the discretization of the domain as shown in Fig. (1). The grid points T_j and T_k are assumed to lie on the interfaces. In addition to the boundary conditions Eq. (4), Eq. (5), we assume the continuity of the temperature and flux at the inner interfaces as

$$\begin{aligned} T_{b^-} &= T_{b^+} \\ k_i\frac{\partial T_{b^-}}{\partial x} &= k_{i+1}\frac{\partial T_{b^+}}{\partial x} \\ T_{c^-} &= T_{c^+} \\ k_i\frac{\partial T_{c^-}}{\partial x} &= k_{i+1}\frac{\partial T_{c^+}}{\partial x} \end{aligned} \tag{6}$$

The temperature dependent blood perfusion in the normal and tumour tissues is based on the results described in Kengne et al. (2012)

$$\begin{aligned} W_{Muscle}(T) &= 0.45 + 3.55\ exp\left(\frac{-(T-45)^2}{12}\right), T \leq 45 \\ &= 4.0 \qquad\qquad\qquad\qquad\quad , T > 45 \end{aligned} \tag{7}$$

$$\begin{aligned} W_{Tumour}(T) &= 0.833, && T < 37 \\ &= \frac{0.833(T-37)^{4.8}}{5438}, && 37 \leq T \leq 42 \\ &= 0.416, && T > 42 \end{aligned} \tag{8}$$

The temperature dependent blood perfusion in the normal and tumor tissues is taken as(given in Lang et al. (1999)) Further, we assume the temperature dependent perfusion in dermis and subcutaneous tissue linearly depends on temperature. That is

$$w_{ds} = \omega_0(1 + \gamma T) \tag{9}$$

where ω_0 and γ denote the normal perfusion and the linear coefficient of temperature dependence.

For the inner points of the discretization, we use the standard first-order time and second-order space finite differencing. That is, the space-order finite differencing at the point j is given by

$$\frac{\partial T_j}{\partial t} = \frac{k_i(T_{j-1} - 2T_j + T_{j+1})}{\Delta x^2} \tag{10}$$

where T_j is the temperature at the spatial point j in the layer i.

The time differencing for the points on the interfaces, we use central difference method as

$$\frac{\partial T_j}{\partial t} = \frac{D_{i+1}\frac{\partial T_{b+}}{\partial x} - D_i\frac{\partial T_{b-}}{\partial x}}{\Delta x} \tag{11}$$

First-order forward and backward differences for the spatial derivative for the model Eq. (2) and using Eq. (7) then give (Hickson et al. (2009), Hickson et al. (2009)

$$\frac{\partial T_j}{\partial t} = \frac{D_{i+1}\left(\frac{T_{j+1}-T_j}{\Delta x}\right) - D_i\left(\frac{T-j-T_{j-1}}{\Delta x}\right)}{\Delta x} + \alpha_1(T_j - T_b) + \alpha_2(T_j - T_b)\left[\alpha(T_j - 37)^{4.8}\right] + Q \tag{12}$$

where $\alpha_1 = \frac{c_b w_b}{\rho c}$, $\alpha_2 = \frac{c_b \rho_b}{\rho c}$ and $Q = \frac{Q_m + Q_h}{\rho c}$

Rearranging Eq. (12), we get

$$\frac{\partial T_j}{\partial t} = \frac{D_{i+1}T_{j+1} - (D_{i+1} + D_i)T_j + D_i T_{j-1}}{\Delta x^2} + \alpha_1(T_j - T_b) + \alpha_2(T_j - T_b)\left[\alpha(T_j - 37)^{4.8}\right] + Q \tag{13}$$

Similarly, for the point T_k we have

$$\frac{\partial T_k}{\partial t} = \frac{D_{i+1}T_{k+1} - (D_{i+1} + D_i)T_k + D_i T_{k-1}}{\Delta x^2} + \alpha_1(T_k - T_b) + \alpha_2(T_k - T_b)\left[\alpha(T_k - 37)^{4.8}\right] + Q \tag{14}$$

The Eqs. (13 & 14) can be solved by using standard integration techniques such as Euler time stepping. For example

$$T(x + \Delta t) = T(t) + \Delta t \chi T(t) \tag{15}$$

where the T and χ are respectively vector and matrix notations.

The mixed boundary condition at L_0, given by Eq. (16), is included in the finite difference scheme by adding a fictitious point T_{n+1} at a distance Δx to the right of boundary at L_0 as:

$$\frac{\partial T}{\partial t} = -\frac{q_0}{k} \tag{16}$$

Discretizing Eq. (16), we have

$$\frac{T_{n+1} - T_n}{\Delta x} = -\frac{q_0}{k} \tag{17}$$

Now substituting the value of T_{n+1} from Eq. (17) into the Eq. (14), we get

$$\frac{\partial T_n}{\partial t} = \frac{D_{i+1}(T_n - \frac{q_0 \Delta}{k}) - (D_{i+1} + D_i)T_k + D_i T_{n-1}}{\Delta x^2} + \alpha_1(T_n - T_b) + \alpha_2(T_n - T_b)\left[\alpha(T_n - 37)^{4.8}\right] + Q \tag{18}$$

IV. Discussion

The transport of heat by conduction and by temperature dependent perfusion in the dermal regions of the human body in normal and tumour tissues has been studied through Pennes bioheat equation. The oscillating boundary condition has been incorporated into the model for better estimates of the temperature distribution. The skin has a non-perfused epidermis layer and deep tissue consists of dermis and subcutaneous layers in which the perfusion plays an important role for estimating the variations of temperature distribution. Prediction of spatial temperature distribution and the role of oscillatory heat flux has been studied in this model for normal tissue and tumour tissue. The results were illustrated with the help of graphs obtained by the MATLAB and FlexPDE softwares.

The uncontrolled and replicated growth of tumour cells leads to the un- usual temperature variations in the normal tissues surrounding it. The tumour cells can be damaged by applying a concentrated beam of heat radiations at the site of the source of the tumour called local hyperthermic therapy. The continuous application of the heat may damage the normal tissue cells surrounding the tumour region. It is thus imperative to study the heat distribution in the normal as well as in tumour tissue regions of the human body. A mathematical model based on Pennes bioheat equation has been taken with some appropriate terms for analysing the variation in tissue temperature. MATLAB and Flex- PDE softwares were used for simulation purposes and the analysis of the results were obtained by using finite difference method. The *in vivo* and *in vitro* studies reveal the fact that the response of tissue to the external heat stress is temperature dependent (Gowrishankar et al. (2004), Davies et al. (1977)). For example, in the commonly used clinical hyperthermia, the temperature rise of $41 - 43^0C$ enhances the blood flow significantly (Erdamn et al. (1998)).

V. Conclusion

We solved the model Eq. (2) with the help of finite difference method discussed in the paper and the results were plotted using MATLAB and FlexPDE (2009) softwares. Figs. (5, 6) respectively show the temperature response at dermal region for different values of heating frequencies, $\omega = 0.001$, $\omega = 0.005$ and $\omega = 0.01$. The Figs. (2, 3) show the temperature distribution through the normal muscle tissue and the tumour tissue respectively at time $t = 3600s$ in the absence and in the presence of sinusoidal heat flux. It is clear from the graphs that the sinusoidal heat flux affects the temperature amplitude over the time. Thus, the presence of sinusoidal heat

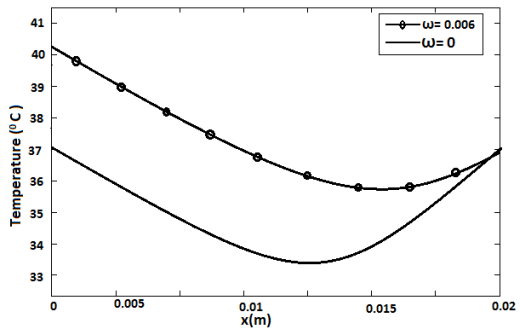

Fig. 2. Temperature distribution across the normal muscle tissue layers at t=1 hr.

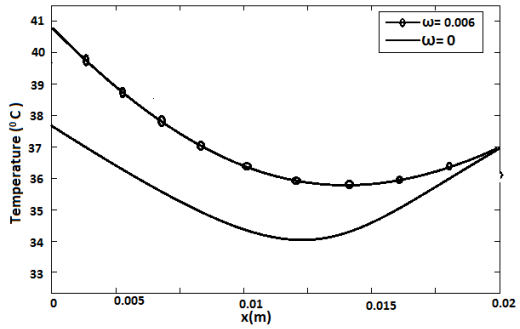

Fig. 3. Temperature distribution across the skin layers in presence of tumour tissue at t=1 hr.

flux diminishes the temperature amplitude along the tissue depth. The peak temperature in the curves shown in Fig. (4) depicts a decrease due to the increase in rate of perfusion by increased temperature. The increase in perfusion rate increases the rate of heat loss from the tissues to the blood.

Figs. (7, 8) show that the arterial blood temperature has no effect on the sinusoidal temperature at the dermal regions for both normal and tumour tissues. Also, we conclude that the average temperature value at the skin surface is approximately equal to the arterial blood temperature. Further, as the arterial blood perfusion raises, the temperature increases rapidly.

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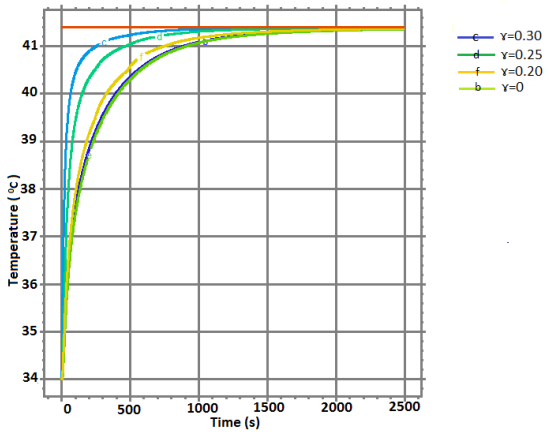

Fig. 4. Temperature of skin at the surface for different temperature coefficients.

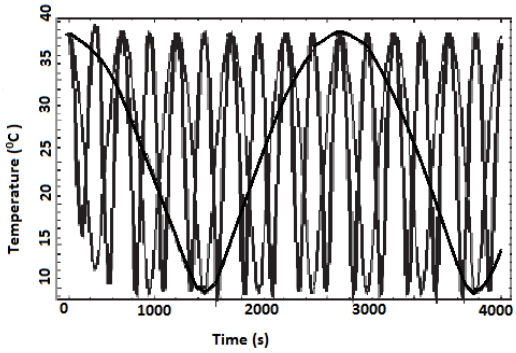

Fig. 5. Tissue response at the skin surface in presence and absence of sinusoidal heat flux for the normal muscle tissue.

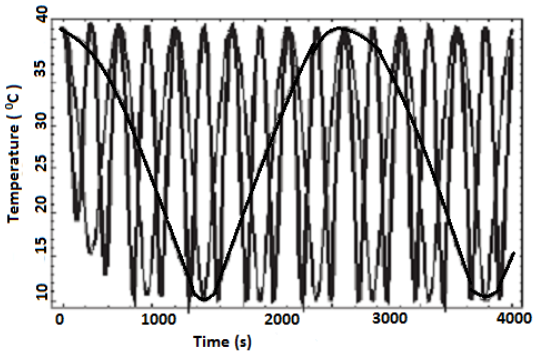

Fig. 6. Tissue response at the skin surface in presence and absence of sinusoidal heat flux for the tumour tissue.

References

Babita, K., & Neeru, A. (2018). Three dimensional finite difference model to study the thermal stress in peripheral regions of human limbs immediately after physical exercise, Internatinal Journal of Pharma Medicine and Biological Sciences,7(1),doi:10.18178/ijpmbs.7.1.1-7.

Davies, C. R., Saidel, G. M., & Harasaki, H. (1977). Sensitivity analysis of one dimensional heat transfer in tissue with temperature dependent perfusion, J Biomech Eng, 119, 77-80.

Dai, W., Bejan, A., Tang, X., Zhang, L., & Nassar, R. (2006). Optimal temperature distribution in a three dimensional triple-layered skin structure with embedded vasculature, Journal of Applied Physiology, 99, 104702-1-104702-9.

Diller, K. R., & Hayes, L. J. (1983). A finite element model of burn injury in blood perfused skin, Journal of Biomedical Engineering, 105, 300-307.

El-dabe, T. M. N., Mohamed, M. A. A., & El-Sayed, A.

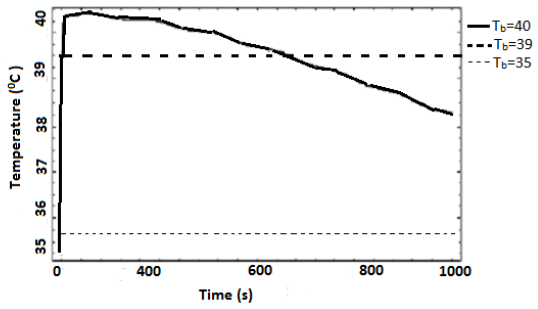

Fig. 7. Effect of arterial blood temperature on the temperature response at the skin surface in the normal tissues.

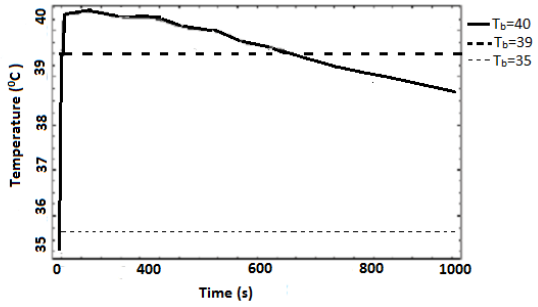

Fig. 8. Effect of arterial blood temperature on the temperature response at the skin surface in the tumour tissues.

F.(2003). Effects of microwave heating on the thermal states of biological tissues, African Journal of Biotechnology, 2(11), 453-459.

Erdmann, E., Lang, J., & Seebass, M. (1998). Optimisation of temperature distributions for regional hyperthermia based on a nonlinear heat transfer model, Ann NY Acad Sci, 858, 36-46.

Frahm, K. S., Andersen, O. K., Arendt-Nielsen, L., & Morch, C. D.(2010). Spatial temperature distribution in human hairy and glabrous skin after infrared CO2 laser radiation, BioMedical Engineering OnLine,9, 1-9.

Gowrishankar, T. R., Stewart, D. A., Martin, G. T., &. Weaver, J. C.(2004). Transport lattice models of heat transport in skin with spatially heterogeneous, temperature-dependent perfusion, Biomedical Engineering Online, 3(42), doi:10.1186/1475-925X-3-42.

Hall, J.E., & Guyton, A.C.(2012). A Text Book of Medical Physiology, Elsevier Sciences, 12th edi.

Hickson, R., Barry, S., & Mercer, G.(2009). Critical times in multilayer diffusion. Part-1: exact solutions, International Journal of Heat and Mass Transfer, 52(25-26), 5776-5783.

Hickson, R., Barry, S., & Sidhu, H.(2009) Critical times in one-and two-layered diffusion, Australian journal of Engineering Education, 15(2), 77-84.

Kamangar, S., Khan, M. A., Badruddin, I. A., Yunus Khan, T. M., & Ghazali, N. N., (2019). Numerical Analysis of heat transfer in human head, Journal of Mechanical Science and Technology, 33, 3597-3605. http://doi.org//10.1007/s12206-019-0654-x.

Kengne, E., Lakhssassi, A., & Vaillancourt, R.(2012). Temperature distribution in living tissue simultaneously subjected to oscillatory surface and spatial heating: Analytical and numerical analysis, International Mathematical Forum, 7(48), 2373-2392.

Khalid, N., Khanday, M. A., & Ganie, B. A. (2017). 2D model on heat regulation in human body with dermal tumour , International conference on Applications of Mathematics to Non-linear Sciences, Electronic Journal of Differential equations, conference 24 (pp. 75-83): URL:http://ejde.txstate.edu.

Khanday, M. A., & Fida, H.(2015). Explicit formula of finite difference method to estimate human peripheral tissue temperature during exposure to severe cold stress, Journal of Thermal Biology, 48, 51-55.

Lang, J., Erdmann, B., & Seebass, M.(1999).Impact of nonlinear heat transfer on temperature control in regional hyperthermia, IEEE Trans. on Biomedical Engineering, 11291138.

Park, D., Seung, N.H., Kim, J., Jeong, H. & Lee, S.(2007) Infrared thermography in the diagnosis of unilateral carpal tunnel syndrome, Muscle Nerve, 36, 575576.

Pennes, H. H.(1948). Analysis of tissue and arterial blood temperature in the resting forearm, Journal of Applied Physiology, 1, 93-102.

PDE Solutions, (2009). Inc. FlexPDE,Spokane Valley, Version 6.36 http:// www.pdesolutions.com.

Shih, T. C., Yuan, P., Lin, W. L., & Kou, H.S.(2007). Analytical analysis of the Pennes bioheat transfer equation with sinusoidal heat flux condition on skin surface, Medical Engineering and Physics, 29,946953.

Shusterman, V., Anderson, K.P., & Barnea, O.(1997). Spontaneous skin temperature oscillations in normal human subjects, Am. J. Physiol. Regul. Integr. Comp. Physiol., 273, R1173R1181.

Stoll, A. M.(1977). Thermal properties of human skin related to non-destructive measurement of epidermal thickness, J Invest Dermatol, 69, 328-332.

Stoll, A. M., Chianta, M. A., & Piergallini, J.R.(1979). Thermal conduction effects in human skin, Aviation space Env Med, 50, 778-787.