Volume 65, Issue 3, 2021
Journal of Scientific Research
Institute of Science,
Banaras Hindu University, Varanasi, India.

The Beauty of Fractal Patterns in Geology

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Abstract: **Marked presence of the classical geometric patterns to floral motifs all around us has inspired the scientific community to establish common ground between the glimpses of nature's beauty and the abstract mathematics. Such fascinating self-repeating patterns are referred to as Fractals. The natural elegant patterns frequently mentioned as the finger mark of God bring unique inventiveness of nature to vanguard. Interestingly, Ammonites possessing shell patterns similar to the modern snails follow the Fibonacci sequence. Like-wise fish fossils also exhibit fractal patterns that bear a resemblance to tree branches and hyperbolic graphs. The spectacular suture lines in the fossils or the Golden mean in the Nautilus shell or the fractal-like nature of hyperbolic groups that imitate the symmetry of the fish fossils; they all provide an insight into the mathematical definition of symmetry in understanding how nature works. The ubiquitous presence of the divine proportion (φ =1.618) among plants, fossils and even cosmos remains no less astounding. There is a minuscule design at each level and a grand design above all. It is strongly felt that lessons of timeless beauty of universal science of fractals will excite spark among young geologists to look at the sacred geometrical patterns hidden in the geological fossils.**

Index Terms: **Fractals, Fibonacci numbers, Golden Ratio, Self-repeating patterns, Cambrian Explosion, Ammonites.**

I. INTRODUCTION

One of the big mysteries of science states that beneath the complexities of the universe there is an underlying harmonious order, whose existence Einstein described as "a miracle". No less astonishing, the fundamental laws of universe can be written in the language of Mathematics (Farmelo, 2019). Impeccable documentation with flawless precision has been conserved by nature along the geological time scale (Mohan and Prashant, 2013). These self-repeating patterns are described as Fractals (Mohan and Prashant, 2013). The planets encircle the stars just like the electrons encircle the neutrons. Stars clump together in the same way as the in galaxies atoms clump together in molecules. *Ansel Adams* said, "I believe the world is incomprehensibly beautiful, an endless prospect of magic and wonder". In fact, fractals have changed the way we understand nature. They are the most beautiful and extremely powerful creation of nature. This was further substantiated by *Plato* who described the Beauty as, "is always an aspect of the good," and "in the beautiful and good, proportion is involved." Fractals contain self-similar patterns of complexity that increases with magnification. Famous mathematician *Benoit Mandelbrot* known for his contribution to fractal geometry, coined the term fractal and developed a theory of "roughness and self-similarity" in nature (Linton, 2021). When a fractal pattern is divided to parts we obtain almost an identical reduced-size copy of the whole (http://www.marywood.edu/math/fractals.html). The limitless complexities expressed through such equations reflect the unparallel mathematical beauty of fractals. The creation of self–affine patterns that are nested within one another is inherent in the geometry of fractals. It is this tenet of oneness cardinal to all the geometrical patterns that pervades the architectonics. Fractals bring creativity of nature to the forefront because they are referred to as the thumb print of God.

An attempt has been made in this paper to present a snapshot of fractals which throw fresh look at familiar subjects in Geology in general and Paleontology in particular. Fractals offer the natural mathematical language to describe much of what the geologists observe. The geological records imply that there are abundant small events, occurring close to one another space and time, with fewer large events occurring in the same temporal or spatial region. Such relations are indicative of power law scaling. This hints that fractal geometry is useful for quantifying geological patterns (Barton and La, Pointe, 1995). Taking cognizance of the fact that ca. 540 Ma there apparently occurred a world-wide dramatic acceleration in the rate of evolution: the "Cambrian Explosion". These hallmarks were captured by Paleontologists to study the history of life on Earth as based on the fossil record. Paleontology, an important facet of geology, is endowed with information about the anatomies, ecologies,

DOI: 10.37398/JSR.2021.650302

physiologies, and spatial and temporal dynamics of past life. Since the recognition of fossils as remnants of once-living organisms by early polymaths such as Nicholas Steno, paleontology has become the pellucid window into the long history of life on Earth. However, Modern paleontologists use a variety of tools to describe the fossils remains of plants and animals preserved in rock - a blueprint of amazing patterns with immaculate perfection.

II. Ammonites: a form perfected by nature

The observational drawings about spiral growth by *Leonardo da Vinci's* were directed to "understand the forces and processes underlying their forms" establish the connection between nature and mathematics. The spirals are omnipresent in nature, from the micro to the macro scale. Potential examples from fossils kingdom describe legendary saga of evolution in terms of repeating patterns are elaborated here. It is noteworthy that the shell design of extinct Cephalopod accomplished exquisiteness in the geometry of the shell size, regularity of shell coiling and ornamentation. The shape of the Cephalopod is aesthetically pleasing because of the mathematically proportional dimensions. The lobes and saddles in them are fluted and the subdivisions are generally rounded rather than of saw-toothed. Ammonoids (Fig.1) of this type are paramount species from a biostratigraphical perspective (https://es.qaz.wiki/wiki/Ammonoidea).

Fig. 1

Amazingly, the ammonites range from a few inches to several feet! Each shell is divided into many different chambers. Ammonites were aquatic creatures, and controlled the air pressure inside their shells to maintain their buoyancy. Ammonite shells are thus naturally occurring example of the Fibonacci sequence.

With the passage of time, the suture lines in the late Jurassic Ammonite fossils have the propensity to become intricate practically similar to the fern leaf (Fig.2). This type of suture is characteristic of Jurassic and Cretaceous Ammonoids, but extends to the Permian as well. These fern like shapes are called fractals, and have the property of reflecting even a tiny piece of its shape as the whole thing when it is magnified. The suture lines of Ammonite are evocative of a fractal: 'a splintered geometric shape that can be broken into segments and each one is a reduced-size copy of the whole'. In other words they are a self-repeating pattern reducing in size.

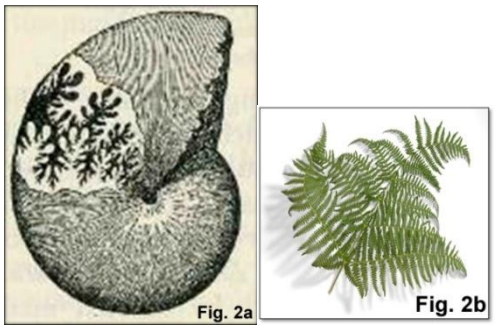

Fig. 2a Fig. 2b

Mathematics is queen of Sciences because it reveals Nature's deepest secrets. The blossoming relationship between mathematics and nature preserved in these fossils for millions of years has been remarkably portrayed the beauty in time, creation and computation. "Nature gives to every time and season some beauties of its own"- *Charles Dickens*.

Fibonacci sequence, a series of numbers was developed by a mathematician *Leonardo Bonacci* in the 13th century. The Ammonite shell is formed in a Fibonacci spiral (Fig.3).

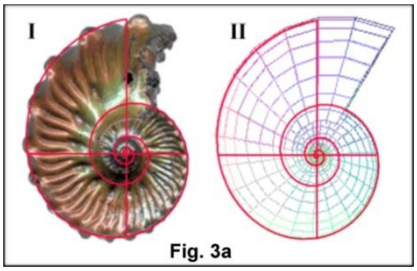

Fig. 3a

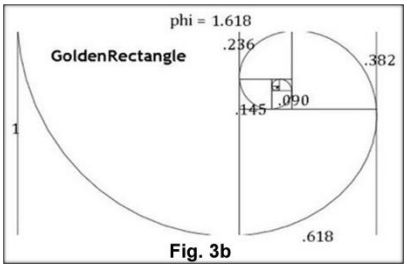

Fig. 3b

Similar spirals can be observed in nature such as in galaxies, the arrangement of leaves around a stem and the arrangement of a pinecone etc. While quantifying these observations, mathematics rolls down its expression because the Universe speaks to us in numbers. These spirals conform to an explicit mathematical equation: $Fn = Fn\text{-}_1 + Fn\text{-}_2$. Each number in the sequence is the sum of the previous two. A series of squares with lengths equal to the Fibonacci numbers (1,1,2,3,5, etc) can be

constructed and a line along the diagonals of each square would form a Fibonacci spiral. This Golden Ratio or commonly known as "φ or Phi" is an irrational number whose decimal place is never-ending and non-repeating (1.618….to ∞).

"In all things of nature there is something of the marvelous – *Aristotle*". The shell of Nautilus having unblemished design embodies the Golden ratio. Likewise, the shells of Mollusk grow in a logarithmic spiral fashion to showcase the beauty of the Fibonacci sequence. The ratio of the short side versusu the long length is consistently 1: $(1+\sqrt{5})/2 \rightarrow$ 1:1.618. The logarithmic spiral of the Fibonacci sequence instantly allures to the eye, and it has mathematical attraction as well.

The figure 4 shows the construction of a golden spiral and its nearest match in nature. Cross-sections of a Nautilus shell show the spiral curve of the shell. The shell has grown by a factor of the Golden ratio in one turn. The golden logic is pervasive in the Universe from the cosmological to the sub-atomic scale. This golden ratio explains the foundation of how nature brings a balance. Likewise, the fish fossils offer captivating mathematical perception into fractals that are entwined with both; the Chaos and Group theories. The branching pattern a fin of a fish corresponds to a fractal, a mathematical and visual pattern that infinitely repeats itself. Fish fossils contain naturally-occurring fractals (Harrison, 2017). The Chaos theory and fractals subsume complex objects and the chaotic activities frequently assemble fractal patterns. The examination of fish fins through drawing guides a novel awareness into patterns within them and their relationship to mathematics. For instance, the fractals of hyperbolic groups imitate the symmetries spotted in the fish fossils. This further allows clear understanding of the common ground of group theory and the mathematical characterization of symmetry. Consequently, the Fibonacci sequence in Ammonites and fractals in fish fins through drawings discover the intrinsic appeal of patterns and symmetry looming in architecture, art and natural world. Over a span of time, obsession among artists has flourished to value symmetry and repetition in design. The Fibonacci spirals and fractals detected in fossil kingdom allure broad range viewers who attempt to appreciate rationally the entirely of God's creation.

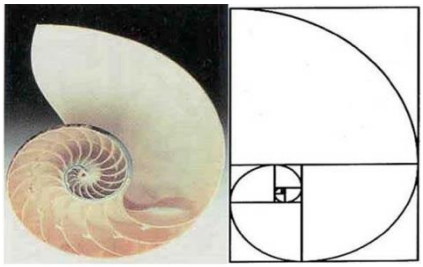

Fig. 4

Numerous Graphoglyptids and Chondritids have fractional dimensionality and self-resemblance over a notable gamut of quantifying scales (Fig.5). Intriguingly, such "fractal traces" are amid the most reckoned trace fossils of the resurgence (Baucaon, 2010).

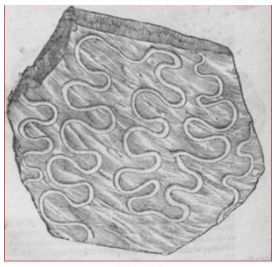

Fig. 5

CONCLUSION

Fractals are classified structures and their entire geometric structure can be considered as an articulation of self-sustaining processes bringing out connections between different orders of scale. Let all of us with the budding geologists explore the embedded fractals – the integral members of the macrocosm--and appreciate them before they perish unnoticed. "Nature is painting for us, day after day infinite beauty" -*John Ruskin*. This note should excite the spark amongst young geologists with never ending quest to look at geological problems in innovative ways. Palaeontology studded with laudable advances whispering secrets of evolution is no longer seen as monotonous subject. To sum up, there is a minuscule design at each level and a grand design above all.

ACKNOWLEDGEMENT

This paper has benefited from numerous sites browsed through the internet web/blogs/Wikipedia (http://www.marywood.edu/math/fractals.html;https://es.qaz.wiki/wiki/Ammonoidea;www.nationalgeographic.org; www.frontiersin.org; www.pnas.org) pertaining to self-affine patterns which are gratefully acknowledged. The recent book by Prof. Linton enlarged the scope of thinking on the application of fractals. Prof. Karmeshu, Prof. of Mathematics (Retd.) JNU is thanked with gratitude for helpful suggestions.

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