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Modified Classes of Regression Type Exponential Estimators of Population Mean

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Abstract: In this paper, classes of modified regression--type exponential estimators for estimating population means using known parameters of auxiliary variable have been suggested. The biases and mean square errors of the suggested classes of estimators have been derived up to first order of approximation and theoretical conditions for which the suggested estimators are more efficient than other existing estimators considered in the study have also been established. Numerical illustration has been conducted and the results revealed that the suggested estimators are more efficient than existing estimators even for extreme cases of correlation coefficient.

Index Terms: **Auxiliary variable, bias, efficiency, mean square error, population mean**

I. Introduction

The estimation of population mean of study variable with greater precision is an unrelenting issue in sampling theory and the precision of estimates can be improved by increasing the sampling size, but doing so tend to sabotage the benefits of sampling. Therefore, the precision may be increased by using an appropriate estimation procedure that utilizes auxiliary information which is closely related to the study variable therefore ratio and regression estimators came into existence. Many contributions over modification of these conventional estimators are present in literature. These include Tracy and Singh (1999), Singh and Kumar (2011), Singh and Audu (2015), Audu et al (2020) and Ahmed et al. (2016). Bahl and Tuteja (1991) introduced ratio and product-type exponential estimators which were later extended by various authors including Singh et al (2008), Singh and Ahmed (2015), Ahmed and Singh (2015) and Singh et al (2020).

Difference-cum-ratio-type estimators were suggested by Kadilar and Cingi (2004) by using coefficient of variation and kurtosis of auxiliary variable X and modified by Kadilar and Cingi (2006) by adding correlation coefficient. Later on, extended subsequently by Subramani and Kumarapandiyan (2012), Abid et al (2016), Subzar et al (2017), Subzar et al. (2018a), Subzar et al (2018b), Subzar et al (2018c) by using known parameters of auxiliary variable . Recently, Yadav and Zaman (2021) suggested some class of estimators by using conventional and non-conventional location parameters of auxiliary variable and sample size and found that their estimators were more efficient than the estimators developed by above mentioned authors. In this paper, exponential-based estimators have been suggested to enhance the efficiency of Yadav and Zaman (2021) estimators as well as all other estimators mentioned in this para.

2. Notations used

Let study character and auxiliary character be denoted by Y and X respectively. Y_i and X_i be the value of population units for i= 1, 2, 3, …, N . Let us further denote:

$\bar{Y}$ and $\bar{X}$ be the population means of study and auxiliary characters respectively.

S_Y^2 and S_X^2 be the population mean squares of Study and auxiliary characters respectively.

C_Y and C_X be the coefficients of variation of study and auxiliary characters respectively for the population.

$QD = \dfrac{Q_3 - Q_1}{2}$ be the quartile deviation of auxiliary character X for the population.

HL = $\text{median}\left(\dfrac{X_j + X_k}{2}, 1 \le j \le k \le N\right)$ Hodges –

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Lehmann Estimator.

$MR = \dfrac{X_{(1)} + X_{(N)}}{2}$ be the range of auxiliary character X for the population.

$DM = \dfrac{D_1 + D_2 + ... + D_9}{9}$ be the decile Mean for auxiliary character X for the population.

$G = \dfrac{4}{N-1}\sum_{i=1}^{N}\left(\dfrac{2i-N-1}{2N}\right)X_{(i)}$ be the Gini's Mean Difference for auxiliary character X.

$D = \dfrac{2\sqrt{\pi}}{N-1}\sum_{i=1}^{N}\left(i-\dfrac{N+1}{2N}\right)X_{(i)}$ be the Downton's Method for auxiliary character X. and $S_{pw} = \dfrac{\sqrt{\pi}}{N^2}\sum_{i=1}^{N}\left(2i-N-1\right)X_{(i)}$ be the probability Weighted Moment of auxiliary character X .

3. Existing Estimators

Kadilar and Cingi (2004) suggested modified estimators for estimating population mean in simple random sampling as;

$$t_{ai} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(a_{i1}\bar{x} + a_{i2}\right)}\left(a_{i1}\bar{X} + a_{i2}\right),\ i = 2,3,4,5$$

$$\text{and for } i = 1 \quad t_{a1} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\bar{x}} \tag{1}$$

where $a_{21} = 1, a_{22} = C_X;\ a_{31} = 1, a_{32} = \beta_2;$
$a_{41} = \beta_2,\ a_{42} = C_X;\ a_{51} = C_X, a_{52} = \beta_2$

The biases and the MSEs of t_{ai} are given by,

$$B(t_{ai}) = \frac{1-f}{n} R_{ai}^2 \frac{S_X^2}{\bar{Y}},\ i = 1,2,......5 \tag{2}$$

$$MSE(t_{ai}) = \frac{1-f}{n}\left\{R_{ai}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right\}, \tag{3}$$
$$i = 1,2,......5$$

where $R_{a1} = \dfrac{\bar{Y}}{\bar{X}} = R,\ R_{a2} = \dfrac{\bar{Y}}{\left(\bar{X} + C_X\right)}, R_{a3} = \dfrac{\bar{Y}}{\left(\bar{X} + \beta_2\right)},$

$R_{a4} = \dfrac{\bar{Y}\beta_2}{\left(\bar{X}\beta_2 + C_x\right)},\ R_{a5} = \dfrac{\bar{Y}C_X}{\left(\bar{X}C_X + \beta_2\right)}$

Kadilar and Cingi (2006) suggested an improvement over Kadilar and Cingi (2004) estimators by using correlation coefficient as;

$$t_{bi} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(b_{i1}\bar{x} + b_{i2}\right)}\left(b_{i1}\bar{X} + b_{i2}\right) i = 1,2,3,4,5 \tag{4}$$

where $b_{11} = 1, b_{12} = \rho;\ b_{21} = C_X, b_{22} = \rho;\ b_{31} = \rho,$
$b_{32} = C_X;\ b_{41} = \beta_2, b_{42} = \rho;\ b_{51} = \rho, b_{52} = \beta_2$

The biases and MSEs of the estimators are respectively given by,

$$B(t_{bi}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}} R_{bi}^2, i = 1,2,.....,5 \tag{5}$$

$$MSE(t_{bi}) = \frac{1-f}{n}\left[R_{bi}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right] i = 1,2,3,4,5 \tag{6}$$

where, $R_{b1} = \dfrac{\bar{Y}}{\left(\bar{X} + \rho\right)}, R_{b2} = \dfrac{\bar{Y}C_X}{\left(\bar{X}C_X + \rho\right)},$

$R_{b3} = \dfrac{\bar{Y}\rho}{\left(\bar{X}\rho + C_X\right)}, R_{b4} = \dfrac{\bar{Y}\beta_2}{\left(\bar{X}\beta_2 + \rho\right)}, R_{b5} = \dfrac{\bar{Y}\rho}{\left(\bar{X}\rho + \beta_2\right)}$

Subramani and Kumarpandiyan (2012) improved Kadilar and Cingi (2004, 2006) estimators using co-efficient of skewness and median of the auxiliary variable X as:

$$t_{ci} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(c_{i1}\bar{x} + c_{i2}\right)}\left(c_{i1}\bar{X} + c_{i2}\right) i = 1,2,3 \tag{7}$$

$c_{11} = \beta_1, c_{12} = M_d;\ c_{21} = 1, c_{22} = M_d;\ c_{31} = C_X, c_{32} = M_d$

The biases and MSEs of the estimators are respectively given by

$$B(t_{ci}) = \frac{1-f}{n}\frac{S_X^2}{\bar{Y}} R_{ci}^2 \tag{8}$$

$$MSE(t_{ci}) = \frac{1-f}{n}\left[R_{ci}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right] i = 1,2,3 \tag{9}$$

where $R_{C1} = \dfrac{\bar{Y}\beta_1}{\left(\bar{X}\beta_1 + M_d\right)}, R_{c2} = \dfrac{\bar{Y}\rho}{\left(\bar{X} + M_d\right)}, R_{c3} = \dfrac{\bar{Y}c_X}{\left(\bar{X}c_X + M_d\right)}$

Abid et al (2016) suggested improved ratio- type estimators for estimating the population mean using measure of dispersion of auxiliary variable X as:

$$t_{di} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(d_{i1}\bar{x} + d_{i2}\right)}\left(d_{i1}\bar{X} + d_{i2}\right) i = 1,2,3 \tag{10}$$

$d_{11} = 1, d_{12} = DM;\ d_{21} = C_X, d_{22} = DM;\ d_{31} = \rho, d_{32} = DM$

The biases and the MSEs of the class are respectively given by,

$$B(t_{di}) = \frac{1-f}{n}\frac{S_X^2}{\bar{Y}}R_{di}^2,\ \ i=1,2,3 \quad (11)$$

$$MSE(t_{di}) = \frac{1-f}{n}\left\{R_{di}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right\}, i=1,2,3 \quad (12)$$

$$R_{d1} = \frac{\bar{Y}}{\left(\bar{X}+DM\right)}, R_{d2} = \frac{\bar{Y}C_X}{\left(\bar{X}C_X+DM\right)}, R_{d3} = \frac{\bar{Y}\rho}{\left(\bar{X}\rho+DM\right)}$$

Subzar et al (2017) suggested some new estimators for estimating the population mean using conventional location parameters of auxiliary variable X as:

$$t_{ei} = \begin{cases} \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}QD+D_j\right)}\left(\bar{X}QD+D_j\right),\ i=j=1,2,\dots,9 \\ \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}D_j+QD\right)}\left(\bar{X}D_j+QD\right), i=9+j\ and\ j=1,2,\dots,9 \end{cases} \quad (13)$$

The biases and MSEs of the class are respectively given by,

$$B(t_{ei}) = \frac{1-f}{n}\frac{S_X^2}{\bar{Y}}R_{ei}^2 \quad (14)$$

$$MSE(t_{ei}) = \frac{1-f}{n}\left\{R_{ei}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right\} \quad (15)$$

where, $R_{ei} = \dfrac{\bar{Y}QD}{\bar{X}QD+D_j}$, $i=j=1,2,\dots,9$

and $R_{ei} = \dfrac{\bar{Y}D_j}{\bar{X}D_j+QD}$ $i=9+j,\ j=1,2,\dots,9$

Subzar et al (2018 a) improved Subzar et al (2017) estimators by suggesting a family of ratio-type estimators as:

$$t_{fi} = \begin{cases} \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}+\phi_j\right)}\left(\bar{X}+\phi_j\right), i=j=1,2,\dots,6 \\ \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}\rho+\phi_j\right)}\left(\bar{X}\rho+\phi_j\right), i=6+j\ \&\ j=1,2,\dots,6 \\ \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}C_X+\phi_j\right)}\left(\bar{X}C_X+\phi_j\right), i=12+j\ \&\ j=1,2,\dots,6 \end{cases} \quad (16)$$

Biases and MSEs of t_{fi} are respectively given by

$$B(t_{fi}) = \frac{1-f}{n}\frac{S_X^2}{\bar{Y}}R_{fi}^2 \quad (17)$$

$$MSE(t_{fi}) = \frac{1-f}{n}\left\{R_{fi}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right\}, i=1,2,\dots,18 \quad (18)$$

where, $R_{fi} = \dfrac{\bar{Y}}{\bar{X}+\phi_j}, i=j=1,2,\dots,6;$

$R_{fi} = \dfrac{\bar{Y}\rho}{\bar{X}\rho+\phi_j}, i=6+j\ \&\ j=1,2,\dots,6$

and $R_{fi} = \dfrac{\bar{Y}C_X}{\bar{X}C_X+\phi_j}, i=12+j\ \&\ j=1,2,\dots,6$

$$\phi_1=\left(M_d\times M\right), \phi_2=\left(QD\times TM\right), \phi_3=\left(M_d\times HL\right),$$
$$\phi_4=\left(QD\times HL\right), \phi_5=\left(M_d\times MR\right), \phi_6=\left(QD\times MR\right)$$

Subzar et al (2018 b) suggested another set of ratio-type estimators as:

$$t_{gi} = \frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(g_{i1}\bar{x}+g_{i2}\right)}\left(g_{i1}\bar{X}+g_{i2}\right)\ i=1,2,3,4,5,6 \quad (19)$$

$$g_{11}=\beta_1, g_{12}=TM; g_{21}=\beta_1, g_{22}=MR; g_{31}=\beta_1, g_{32}=HL,$$
$$g_{41}=\beta_2, g_{42}=TM; g_{51}=\beta_2, g_{52}=MR, g_{61}=\beta_2, g_{62}=HL$$

The biases and MSEs of the class are respectively given by,

$$B(t_{gi}) = \frac{1-f}{n}\frac{S_X^2}{\bar{Y}}R_{gi}^2, i=1,2,\dots,6 \quad (20)$$

$$MSE(t_{gi}) = \frac{1-f}{n}\left\{R_{gi}^2 S_X^2 + S_Y^2\left(1-\rho^2\right)\right\} \quad (21)$$

Where, $R_{g1} = \dfrac{\bar{Y}\beta_1}{\bar{X}\beta_1+TM}, R_{g2} = \dfrac{\bar{Y}\beta_1}{\bar{X}\beta_1+MR}, R_{g3} = \dfrac{\bar{Y}\beta_1}{\bar{X}\beta_1+HL},$

$R_{g4} = \dfrac{\bar{Y}\beta_2}{\bar{X}\beta_2+TM}, R_{g5} = \dfrac{\bar{Y}\beta_2}{\bar{X}\beta_2+MR}, R_{g6} = \dfrac{\bar{Y}\beta_2}{\bar{X}\beta_2+HL}$

Subzar et al (2018 c) improved Subzar et al (2018 a) and Subzar et al (2018 b) estimators and suggested efficient estimators for estimating the population mean as:

$$t_{hi} = \begin{cases} \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}+\psi_j\right)}\left(\bar{X}+\psi_j\right), i=j=1,2,\dots,6 \\ \dfrac{\bar{y}+b\left(\bar{X}+\bar{x}\right)}{\bar{x}\rho+\psi_j}\left(\bar{X}\rho+\psi_j\right), i=6+j\ \&\ j=1,2,\dots,6 \\ \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}C_X+\psi_j\right)}\left(\bar{X}C_X+\psi_j\right), i=12+j\ \&\ j=1,2,\dots,6 \end{cases} \quad (22)$$

The biases and MSEs of t_{hi} are respectively given by

$$B(t_{hi}) = \frac{1-f}{n}\frac{S_X^2}{\bar{Y}}R_{hi}^2 \quad (23)$$

$$MSE(t_{hi})=\frac{1-f}{n}\left\{R_{hi}^2S_X^2+S_Y^2\left(1-\rho^2\right)\right\},i=1,2,...,18 \quad (24)$$

where, $R_{hi}=\dfrac{\bar{Y}}{\bar{X}+\psi_j},i=j=1,2,...,6;$

$$R_{hi}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_j},i=6+j\,\&\,j=1,2,...,6$$

$$R_{hi}=\frac{\bar{Y}C_X}{\bar{X}C_X+\psi_j},i=12+j\,\&\,j=1,2,...,6$$

$$\psi_1=(M_d+G),\psi_2=(M_d+D),\psi_3=(M_d+S_{pw}),$$
$$\psi_4=(QD+G),\psi_5=(QD+D),\psi_6=(QD+S_{pw})$$

Yadav and Zaman (2021), suggested the following class of estimators of population mean using some conventional and non-conventional parameters of auxiliary variable along with the information on the size of the sample as,

$$t_{pi}=\begin{cases}\dfrac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\eta_j)}(\bar{X}+\eta_j),i=j=1,2,...,8\\ \dfrac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\eta_j)}(\bar{X}\rho+\eta_j),i=8+j\,\&\,j=1,2,...,8\\ \dfrac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_X+\eta_j)}(\bar{X}C_X+\eta_j),i=16+j\,\&\,j=1,2,...,8\end{cases} \quad (25)$$

The bias and MSE of t_{pi} are given as

$$B(t_{pi})=\frac{1-f}{n}\frac{S_X^2}{\bar{Y}}R_{pi}^2 \quad (26)$$

$$MSE(t_{pi})=\frac{1-f}{n}\left\{R_{pi}^2S_X^2+S_Y^2\left(1-\rho^2\right)\right\},i=1,2,...,24 \quad (27)$$

$$R_{pi}=\frac{\bar{Y}}{\bar{X}+\eta_j},i=j=1,2,...,8;$$

$$R_{pi}=\frac{\bar{Y}\rho}{\bar{X}\rho+\eta_j},i=8+j\,\&\,j=1,2,...,8;$$

$$R_{pi}=\frac{\bar{Y}C_X}{\bar{X}C_X+\eta_j},i=16+j\,\&\,j=1,2,...,8$$

$$\left.\begin{array}{l}\eta_1=(QD\times n),\eta_2=(DM\times n),\eta_3=(TM\times n),\\ \eta_4=(MR+n),\eta_5=(HL\times n),\eta_6=(G\times n),\\ \eta_7=(D\times n),\eta_8=(S_{pw}\times n)\end{array}\right\} \quad (28)$$

4. Proposed Estimators

Having studied above mentioned estimators and motivated by the work of Audu and Singh (2020), the following estimators are proposed.

$$T_k=\begin{cases}\dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}+\eta_j\right)}\left(\bar{X}+\eta_j\right)\exp\left(\left(\dfrac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right),\ k=j=1,2,3...,8\\ \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}\rho+\eta_j\right)}\left(\bar{X}\rho+\eta_j\right)\exp\left(\dfrac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right),\ k=8+j\,\&\,j=1,2,3,...,8\\ \dfrac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}C_X+\eta_j\right)}\left(\bar{X}C_X+\eta_j\right)\exp\left(\left(\dfrac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right),k=16+j\,\&\,j=1,2,3,...,8\end{cases}$$

(29)

Where η_j are same as Yadav and Zaman (2021) given in (28).

5. Bias and MSE of Proposed Estimators

5.1 Bias and MSE of T_k $when\ k=j=1,2,3,...,8$

$$T_k=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}+\eta_j\right)}\left(\bar{X}+\eta_j\right)\exp\left(\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right)$$

To obtain the bias and mean square error T_k let us defined:

$\bar{y}=\left(1+e_0\right)\bar{Y},\bar{x}=\left(1+e_1\right)\bar{X}$ Thus,

$$\left.\begin{array}{l}E(e_0)=E(e_1)=0,E(e_0^2)=\left(\dfrac{1}{n}-\dfrac{1}{N}\right)C_Y^2,\\ E(e_1^2)=\left(\dfrac{1}{n}-\dfrac{1}{N}\right)C_X^2,E(e_0e_1)=\left(\dfrac{1}{n}-\dfrac{1}{N}\right)\rho C_YC_X\end{array}\right\} \quad (30)$$

By substituting the values of $\bar{y}$ and $\bar{x}$ in (29) for $k=j=1,2,3,...,8$ we have

$$T_k=\frac{(\bar{Y}(1+e_0)-b\bar{X}e_1)}{\left(1+\dfrac{\bar{X}e_1}{\bar{X}+\eta_j}\right)}\exp\left(\frac{-\bar{X}e_1}{2\bar{X}+\bar{X}e_1}\right)$$

$$T_k=\left\{\bar{Y}\left(1+e_0\right)-b\bar{X}e_1\right\}\left(1-\pi_je_1+\pi_j^2e_1^2\right)\exp\left(-\frac{1}{2}e_1+\frac{1}{4}e_1^2\right)$$

Where $\pi_j=\dfrac{\bar{X}}{\bar{X}+\eta_j}$

$$T_k=\left(\bar{Y}\left(1+e_0\right)-b\bar{X}e_0\right)\left(1-\pi_je_1+\pi_j^2e_1^2\right)\left(1-\frac{1}{2}e_1+\frac{1}{4}e_1^2\right)$$

$$\begin{aligned}T_k-\bar{Y}&=\bar{Y}e_0-\left\{\bar{Y}\left(\pi_j+\frac{1}{2}\right)+b\bar{X}\right\}e_1\\&+\left\{\left(\pi_j^2+\frac{1}{2}\pi_j+\frac{3}{8}\right)\bar{Y}+\left(\pi_j+\frac{1}{2}\right)b\bar{X}\right\}e_1^2-\left(\pi_j+\frac{1}{2}\right)\bar{Y}e_0e_1\end{aligned} \quad (31)$$

Taking expectation of above and substituting the values from (30), the bias of T_k is obtain as

$$B(T_k)=\frac{1-f}{n}\left[\begin{array}{l}\left\{\left(\pi_j^2+\frac{1}{2}\pi_j+\frac{3}{8}\right)\bar{Y}+\left(\pi_j+\frac{1}{2}\right)b\bar{X}\right\}C_X^2\\-\left(\pi_j+\frac{1}{2}\right)\bar{Y}\rho C_Y C_X\end{array}\right] \quad (32)$$

Squaring (31) both the sides and taking expectation , the MSE of T_i is obtained as:

$$MSE(T_k)=\frac{1-f}{n}\left[\begin{array}{l}\bar{Y}^2C_y^2+\left\{\bar{Y}\left(\pi_j+\frac{1}{2}\right)+b\bar{X}\right\}^2 C_X^2\\-2\bar{Y}\left\{\bar{Y}\left(\pi_j+\frac{1}{2}\right)+b\bar{X}\right\}\rho C_Y C_X\end{array}\right]$$

$$MSE(T_k)=\frac{1-f}{n}\left\{\left(R_{pk}+\frac{R}{2}\right)^2 S_X^2+S_Y^2\left(1-\rho^2\right)\right\} \quad (33)$$

$$\text{where } R_{pk}=\frac{\bar{Y}}{\bar{X}+\eta_j}, k=j=1,2,...,8$$

5.2 Bias and MSE of T_k when $k=8+j\ \&\ j=1,2,3,...,8$

Express (29) in terms of error term e_0 *and* e_1

$$T_k=\frac{(\bar{Y}(1+e_0)-b\bar{X}e_1)}{\left(1+\frac{\rho\bar{X}e_1}{\rho\bar{X}+\eta_j}\right)}\exp\left(\frac{-\bar{X}e_1}{2\bar{X}+_1\bar{X}e_1}\right)$$

$$T_k=\left\{\bar{Y}\left(1+e_0\right)-b\bar{X}e_1\right\}\left(1-\theta_j e_1+\theta_j^2 e_1^2\right)$$

$$\exp\left(-\frac{1}{2}e_1+\frac{1}{4}e_1^2\right), \text{where } \theta_j=\frac{\rho\bar{X}}{\rho\bar{X}+\eta_j}$$

On solving as case I expression for bias and MSE are obtained as

$$B(T_k)=\frac{1-f}{n}\left[\begin{array}{l}\left\{\left(\theta_j^2+\frac{1}{2}\theta_j+\frac{3}{8}\right)\bar{Y}+\left(\theta_j+\frac{1}{2}\right)b\bar{X}\right\}C_X^2\\-\left(\theta_j+\frac{1}{2}\right)\bar{Y}\rho C_Y C_X\end{array}\right] \quad (34)$$

$$MSE(T_k)=\frac{1-f}{n}\left\{\left(R_{pk}+\frac{R}{2}\right)^2 S_X^2+S_Y^2\left(1-\rho^2\right)\right\} \quad (35)$$

5.3 Bias and MSE of T_k $k=16+j\ \&\ j=1,2,...,8$

From (29) $k=16+j\ \&\ j=1,2,...,8$, we have T_k as

$$T_k=\frac{(\bar{Y}(1+e_0)-b\bar{X}e_1)}{\left(1+\frac{C_x\bar{X}e_1}{\bar{X}C_x+\eta_j}\right)}\exp\left(\frac{-\bar{X}e_1}{2\bar{X}+\bar{X}e_1}\right)$$

$$T_k=\left(\bar{Y}\left(1+e_0\right)-b\bar{X}e_1\right)\left(1-\phi_j e_1+\phi_j^2 e_1^2\right)$$

$$\exp\left(-\frac{1}{2}e_1+\frac{1}{4}e_1^2\right), \text{where } \phi_j=\frac{C_X\bar{X}}{C_X\bar{X}+\eta_j}$$

On solving as case I, expression for bias and MSE are obtained as

$$B(T_k)=\frac{1-f}{n}\left[\begin{array}{l}\left\{\left(\phi_j^2+\frac{1}{2}\phi_j+\frac{3}{8}\right)\bar{Y}+\left(\phi_j+\frac{1}{2}\right)b\bar{X}\right\}C_X^2\\-\left(\phi_j+\frac{1}{2}\right)\bar{Y}\rho C_Y C_X\end{array}\right] \quad (36)$$

$$MSE(T_k)=\frac{1-f}{n}\left(\left(R_{pk}+\frac{R}{2}\right)^2 S_X^2+S_Y^2\left(1-\rho^2\right)\right) \quad (37)$$

$$\text{where } R_{pk}=\frac{\bar{Y}C_X}{\bar{X}C_X+\eta_j}, k=16+j\ \&\ j=1,2,...,8$$

6. Theoretical Efficiency Comparisons

In this section theoretical conditions have been obtained under which the suggested estimators are more efficient than the existing estimators under study.

Suggested estimators T_k will be more efficient than Kadilar and Cingi (2004) estimators t_{ai} if

$$MSE(T_k)<MSE(t_{ai})$$

$$\Rightarrow\frac{1-f}{n}\left(\left(R_{pk}+\frac{R}{2}\right)^2 S_X^2+S_Y^2\left(1-\rho^2\right)\right)$$

$$<\frac{1-f}{n}\left[R_{ai}^2S_X^2+S_Y^2\left(1-\rho^2\right)\right]$$

$$\Rightarrow R_{ai}^2-\left(R_{pk}+\frac{R}{2}\right)^2>0\ \forall\, i=1,2,..,5\ \&\ k=1,2,..,24 \quad (38)$$

Thus, the proposed estimators T_k for $k=1,2,..,24$ will be more efficient than the estimators considered by Kadilar and Cingi (2004) for $i=1,2,..,5$ if condition (38) satisfied.

Similarly, we may obtained the theoretical conditions for all other estimators under study.

7. Empirical Study

To compare the efficiency of proposed estimators with existing estimators under study two population data sets have been considered. Population I has taken from Yadav and Zaman (2021) and Population II from Singh and Chaudhary (1986) and given in table 1

Table 1: Parameters of Population Data sets

Parameters	N	N	$\bar{Y}$	$\bar{X}$	ρ	S_Y	S_X	β_2	β_1
Population I	150	40	79.58	6.5833	0.9363	62.1785	4.3564	5.408	1.4984
Population II	34	20	856.4117	208.8823	0.4491	733.1407	150.5059	0.0978	0.9782
Parameters	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
Population I	2	3	4	5	5	6	8	10	13
Population II	70.3	76.8	108.2	129.4	150.0	227.2	250.4	335.6	436.1
Parameters	M_d	TM	MR	HL	QD	G	D	S_{pw}	DM
Population I	5	6	11	7	3	8.2298	9.2542	9.3707	6.22
Population II	150	162.25	284.5	190	80.25	155.446	140.891	199.961	234.82

From the above two population data sets, the MSE of proposed and existing estimators have been calculated. Percentage relative efficiencies (PRE) of the estimators are obtained with respective to sample mean estimator which is given in table 2. The MSE of sample mean estimators for population I and population II are obtained as 70.87969 and 11066.08 respectively.

Table 2: PRE of Proposed Estimators and Existing Estimators

Estimators	PRE		Estimators	PRE		Estimators	PRE	
	Population I	Population II		Population I	Population II		Population I	Population II
t_0	100	100	t_{f5}	760.2296	125.2621	t_{p5}	808.2714	124.9637
Kadilar and Cingi (2004)			t_{f6}	698.408	125.2556	t_{p6}	808.9434	124.8257
t_{a1}	118.9595	66.36868	t_{f7}	693.5764	125.2631	t_{p7}	809.3148	124.7375
t_{a2}	139.7468	66.58353	t_{f8}	588.1806	125.2591	t_{p8}	809.3496	124.9915
t_{a3}	294.5194	66.39789	t_{f9}	717.229	125.2636	t_{p9}	799.6518	124.9267
t_{a4}	122.747	68.52655	t_{f10}	623.9245	125.2606	t_{p10}	808.0055	125.2221
t_{a5}	375.7768	66.40922	t_{f11}	765.5381	125.2642	t_{p11}	807.8063	125.1771
Kadilar and Cingi (2006)			t_{f12}	708.6666	125.2629	t_{p12}	809.8529	125.2355
t_{b1}	148.5621	66.5027	t_{f13}	741.429	125.2606	t_{p13}	808.5702	125.2003
t_{b2}	164.1261	66.5546	t_{f14}	664.1097	125.2502	t_{p14}	809.1621	125.1695
t_{b3}	141.1856	66.84595	t_{f15}	756.8271	125.2617	t_{p15}	809.4889	125.1495
t_{b4}	124.3267	67.72384	t_{f16}	691.9736	125.2541	t_{p16}	809.5196	125.2064
t_{a5}	305.9453	66.4337	t_{f17}	786.1267	125.2634	t_{p17}	805.0069	124.453
Subramani and Kumarapandiyan (2012)			t_{f18}	751.333	125.26	t_{p18}	809.3581	125.1577
t_{c1}	227.67	96.72207	Subzar et al (2018 b)			t_{p19}	809.2561	125.0467
t_{c2}	306.3191	118.1038	t_{g1}	249.586	98.19243	t_{p20}	810.2989	125.191

t_{c3}	358.6303	102.4585	t_{g2}	352.5175	108.4497	t_{p21}	809.6467	125.1036
Abid et al. (2016)			t_{g3}	271.1862	101.1613	t_{p22}	809.9483	125.0282
t_{d1}	319.5059	104.6776	t_{g4}	154.1672	123.8899	t_{p23}	810.1144	124.9796
t_{d2}	408.1064	110.3286	t_{g5}	184.511	124.7678	t_{p24}	810.13	125.1187
t_{d3}	332.179	116.8176	t_{g6}	160.1858	124.2258	Proposed Estimators		
Subzar et al. (2017)			Subzar et al. (2018 c)			T_1	1050.18	188.5983
t_{e1}	139.9046	66.6298	t_{h1}	493.7481	109.271	T_2	1334.323	169.563
t_{e2}	150.6202	66.65388	t_{h2}	512.9846	108.4527	T_3	1322.071	173.6811
t_{e3}	161.4577	66.77006	t_{h3}	515.0823	111.4476	T_4	1499.087	168.0026
t_{e4}	172.3886	66.84837	t_{h4}	451.8484	104.7453	T_5	1372.999	171.713
t_{e5}	172.3886	66.92435	t_{h5}	474.0518	103.5774	T_6	1422.141	174.2787
t_{e6}	183.3858	67.20816	t_{h6}	476.4768	107.8139	T_7	1454.963	175.7661
t_{e7}	205.4791	67.29316	t_{h7}	510.726	119.4146	T_8	1458.325	171.1472
t_{e8}	227.5525	67.60418	t_{h8}	529.9093	118.9861	T_9	1078.358	172.4367
t_{e9}	260.2832	67.96875	t_{h9}	531.9943	120.4869	T_{10}	1356.16	164.729
t_{e10}	166.9132	66.7087	t_{h10}	468.5585	116.8595	T_{11}	1344.302	166.4616
t_{e11}	150.6202	66.68	t_{h11}	490.9685	116.1226	T_{12}	1514.599	164.0618
t_{e12}	142.5704	66.58984	t_{h12}	493.4073	118.6417	T_{13}	1393.523	165.6385
t_{e13}	137.7787	66.55366	t_{h13}	597.5274	114.2211	T_{14}	1440.844	166.7097
t_{e14}	137.7787	66.52829	t_{h14}	614.8387	113.5499	T_{15}	1472.355	167.324
t_{e15}	134.6017	66.47411	t_{h15}	616.687	115.9588	T_{16}	1475.579	165.4002
t_{e16}	130.6518	66.46435	t_{h16}	557.4858	110.3882	T_{17}	1219.736	180.095
t_{e17}	128.2938	66.44008	t_{h17}	579.1112	109.3511	T_{18}	1459.15	167.0781
t_{e18}	126.1255	66.42363	t_{h18}	581.4171	113.0193	T_{19}	1449.42	169.9492
Subzar et al (2018 a)			Yadav and Zaman (2021)			T_{20}	1585.324	165.9813
t_{f1}	682.2668	125.2567	t_{p1}	798.1967	123.8078	T_{21}	1489.543	168.5811
t_{f2}	572.1424	125.237	t_{p2}	807.6308	125.0635	T_{22}	1527.462	170.3631
t_{f3}	707.6131	125.2589	t_{p3}	807.405	124.8595	T_{23}	1552.365	171.3905
t_{f4}	609.01	125.2444	t_{p4}	809.7285	125.1255	T_{24}	1554.897	168.1865

8. Conclusion

In this paper, we have proposed a class of modified regression –type exponential estimators for estimating finite population mean using single auxiliary variable. The mathematical expression for their biases and mean squares have been derived. To illustrate numerically, two population data sets have been considered one with high and one with low correlation coefficient. The result obtained in table 2, revealed that the proposed estimators, $T_1, T_2, T_3, \ldots\ldots, T_{24}$ have higher PRE as compare to the Kadilar and Cingi (2004), Kadilar and Cingi (2006), Subramani and Kumarapandiyan (2012), Abid et al (2016), Subzar et al. (2017), Subzar et al. (2018a), Subzar et al. (2018b), Subzar et al. (2018c), and Yadav and Zaman (2021). This indicates that the proposed estimators are more efficient and can produce better estimate of population mean than that of existing estimators including Yadav and Zaman (2021). Most important aspect of proposed estimators is that they are efficient for high as well as low correlation coefficient between the study variable and auxiliary variable.

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