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Understanding Self and Cross-Efficiencies and Their Relations in Data Envelopment Analysis

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Abstract: **This paper explicitly talks about the relationship between self-efficiency and cross efficiencies. The self-efficiency and cross efficiencies are used in different research works and various secondary goals are also developed using these measurements, but there is no explicit statement or derivation found in any prior art regarding the relationship between self and best cross efficiency for a DMU. Thus, in the present work, the relationship between self and cross efficiencies is examined explicitly and established via some theorems. These relationships can be useful in DEA's further formulations like goal programming models. The paper also complements the author's previous book chapter wherein they have used these relationships. A Numerical illustration is given in the end to establish the results.**

Index Terms: **Data Envelopment Analysis, self-efficiency, Cross-efficiency, Aggressive model, benevolent model**

I. Introduction

The Multi-Criteria Decision-making techniques are developed by the researchers and professionals to assess and evaluate different alternatives and have gained a lot of consideration in various fields to solve real-world problems. With multiple inputs and multiple outputs, for the similar units under evaluation called Decision Making Units(DMUs), Data Envelopment Analysis (DEA) offers a multi-criteria technique for not only their performance evaluation but also benchmarking among themselves. Introduced by Charnes, Cooper, and Rhodes(1978), DEA has expanded its acceptance since then and is used effectively for measuring efficiencies of DMUs. Self-efficiency scores, calculated in the basic DEA models, do not provide a ranking of DMUs and thus often criticized by researchers in various domains. Cross efficiency model is being developed as an alternative for measuring the most efficient units in a peer evaluation mode (Doyle et al.,1994).

Cross-efficiency models form an efficiency matrix where each entry gives an efficiency score of a DMU. A column represents a vector of efficiency values of one DMU with respect to other DMUs under consideration including self as well. The self-efficiency measurements for each DMU are found in the leading diagonal of this matrix.

The cross-efficiency scores are used to find out the average efficiency scores which are named as average peer efficiency evaluation of DMUs. The average is usually the arithmetic mean of the values which is used to form a secondary goal in the basic DEA model (Liang et al., 2008). There may be an aggressive approach to maximize own efficiency and minimize a cross efficiencies of others or a benevolent approach to maximize own efficiency as well as cross efficiencies of others. Several alternative models are developed to measure the cross-efficiency scores based on aggressive as well as benevolent approaches.

The measure of cross-efficiency offers many advantages over the simple measure of efficiency as it provides a unique ranking of DMUs and differentiates among the group of efficient units as well. It can effectively discriminate between good and poor performers. The measure of cross efficiency also eliminates unrealistic weight restrictions schemes in DEA. DEA cross efficiency model gives it powerful discrimination ability and is hence used in various domains. Applications in different fields vary from preference voting (green et al., 1996) to electricity sector (Chen, 2002), economic environmental performance evaluation (Lu and Lo, 2007), Olympic games (Wu et al., 2009), voting model (Soltanifar et al., 2013), and resource allocation (Du et al., 2014) to mention a few.

The self-efficiency and cross efficiencies are used in different research works and various secondary goals are also developed using these measurements. Gupta et al. (2016) also developed goal programming models using trade-offs between self-

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efficiency score and the best cross-efficiency score, wherein self-efficiency is taken as the upper bound and the best-cross efficiency is taken as lower bound for the relative efficiency measurement, yet explicit relationship among self-efficiency and cross-efficiencies are not examined or stated.

This provides need and motivation to examine the basic definition of self and cross efficiency and develop formulations in the form of new theorems to establish relationships between self and cross efficiency scores. This will also be useful in further developments of DEA new models.

The rest of the paper is organized in the following manner. Problem formulation, definitions and objectives are given in section II. Section III gives some of the prior art closely associated with the set problem and the need for the present study to fill in the gap. Section IV proposes some theorems and develops their proofs. Section V illustrates the proposed theorems through a numerical example and conclusions are given in section VI.

II. DEFINITIONS AND OBJECTIVES

According to Charnes et al. (1978), self-efficiency is nothing but the weighted ratio of outputs to the weighted ratio of inputs subject to the definition of basic efficiency which says that the ratio of output by input should be between 0 to 1. We first describe the CCR model as given below.

A. CCR Model

Herein described the self-efficiency DEA model popularly known as Constant Return to Scale model or CCR model given in 1978 by Charnes et al. As it involves several DMUs, say they are n in number. Each of the n DMUs has similar multiple inputs and similar multiple outputs. Let there is total p number of inputs and q number of outputs. So let, for any k^{th} DMU ($k = 1,2,\dots,n$) x_{ik} represents its inputs ($i = 1,2,\dots,p$) and y_{rk} represents its outputs ($r = 1,2,\dots,q$).

Let the t^{th} DMU is under consideration for self-efficiency evaluation. According to the CCR model, to find self-efficiency of the DMU, we need to maximize the weighted ratio of t^{th} DMU's outputs to its inputs subject to the condition that each DMUs weighed ratio should be less than or equal to 1.

The formulation mathematically is given as

$$\text{Maximize} \quad \frac{\sum_{r=1}^{q} \mu_{rt} y_{rt}}{\sum_{i=1}^{p} \vartheta_{it} x_{it}}$$

$$\text{subject to} \quad \frac{\sum_{r=1}^{q} \mu_{rt} y_{rk}}{\sum_{i=1}^{p} \vartheta_{it} x_{ik}} \le 1 \qquad \text{for } k = 1,2,..,n$$

$$\mu_{rt} \ge 0 \qquad \text{for } r = 1,2,..,q$$

$$\vartheta_{it} \ge 0 \qquad \text{for } i = 1,2,..,m$$

Where μ_{rt} and ϑ_{it} are weights of outputs and inputs.

The basic model was given as a fractional programming model. Charnes et al. (1978) further suggested the equivalent formulation in linear form with the substitution as

$$s = \left(\sum_{i=1}^{p} \vartheta_{it} x_{it}\right)^{-1}$$

$$\text{put } u_{rt} = s \times \mu_{rt}$$

$$\text{put } v_{rt} = s \times \vartheta_{rt}$$

Therefore, we get equivalent linear form as

$$\text{Maximize} \quad \sum_{r=1}^{q} u_{rt} y_{rt}$$

$$\text{subject to} \quad \sum_{i=1}^{p} v_{it} x_{it} = 1$$

$$\sum_{r=1}^{q} u_{rt} y_{rk} - \sum_{i=1}^{p} v_{it} x_{ik} \le 0 \qquad for\ k = 1,2,..,n$$

$$u_{rt} \ge 0 \qquad for\ r = 1,2,..,q$$

$$v_{it} \ge 0 \qquad for\ i = 1,2,..,p$$

B. Cross Efficiency Model

1) Cross Efficiency Basic Model

The cross-efficiency for a DMU t with respect to any of the remaining m^{th} DMU is given as

$$\theta_{tm} = \frac{\sum_{r=1}^{q} \mu_{rm} y_{rt}}{\sum_{i=1}^{p} \vartheta_{im} x_{it}} \qquad for\ m = 1,2,..,n$$

Above formula describe that there are n efficiency scores for any DMU t out of which one is its own namely self-efficiency score for $m = t$ and rest $n-1$ are cross efficiency values for $m = 1,2,\dots,n\ m \neq t$. These cross-efficiency values are calculated using the weight of other DMUs in the peer group.

2) Benevolent and Aggressive Model

The two basic models for calculating cross-efficiency scores are known as benevolent and aggressive models. In the benevolent model, efficiency of self is maximized at the same time efficiency of all others is also maximized whereas, in the aggressive model, the efficiency of self is maximized whereas the efficiency of all others is minimized.

The objective of the paper is to examine and give an explicit mathematical interpretation of relationship between all these different efficiencies. Thus, first, we shall show that all cross-efficiencies are greater than equal to zero and then we will establish the relationship between self and cross-efficiencies. Prior to this, development of cross efficiency models and their various applications in previous related research work is discussed in the next section.

III. PREVIOUS RELATED RESEARCH WORK

Cross-efficiency models are developed as an alternative to basic DEA models to provide a ranking for DMUs among other analyses (Doyle et al., 1994). Sexton et al. (1986) presented some critique to basic DEA models and provided some extensions to measuring efficiency as cross efficiency models defining aggressive and benevolent approaches. The traditional DEA model ignores the cooperation relationship among DMUs, so it is difficult to evaluate the DMUs' efficiency reasonably (Chen et al., 2017). The traditional self-evaluation DEA method usually exaggerates the effects of several inputs or outputs of the evaluated DMU, resulting in unrealistic results (Chen et al., 2020).

Doyle and Green (1994) explained cross-efficiency models in a very comprehensive way including their meanings, uses, and derivations. Liang et al. (2008) extended the works of Doyle and Green (1994) and offer various alternative secondary goals in DEA cross-efficiency. Wang et al. (2010) presented more alternative models for cross-efficiency DEA models such as neutral goals. Lim (2012) presented minimax and maximin formulations of finding cross efficiencies in DEA. Several cross-evaluation methods have been proposed to deal with the non-uniqueness of the optimal weights in DEA (Carrillo et al., 2018).

Various applications of these models are found in literature such as Wu et al. (2009), Lu et al. (2007) among others. Chen et al. (2020) proposed a DEA target-setting approach within the cross-efficiency framework via some models and formulations. Mirmozaffari et al. (2017) ranked heart hospitals using cross-efficiency and two-stage DEA. Wu et al. (2020) reviewed the theory and applications of cross-efficiency evaluation. Amin et al. (2021) applied cross-efficiency DEA models for optimization in portfolio selection by use of the alternative optimal solution.

However, none of the prior arts explicitly talks about the relationship between self-efficiency and cross-efficiency scores. It is important to understand these relationships to develop new goal programming models having trade-offs between self and cross-efficiency scores, for example, as used in Gupta et al. (2016). Thus, the present work fills in the gaps in this area and develops new mathematical formulations for their relationships.

IV. Some basic theorems and mathematical formulations

THEOREM 1. *Self-Efficiency is greater than or equal to all cross-efficiencies for any DMU. Mathematically*

$$\theta_{kk} \geq \{\theta_{kj}\} \ \forall j$$

PROOF. self-efficiency for DMU k is given by $\theta_{kk} = \frac{\sum_{r=1}^{q} \mu_{rk} y_{rk}}{\sum_{i=1}^{p} \vartheta_{ik} x_{ik}}$

Where weights μ_{rk} and ϑ_{ik} are obtained as optimal solution of the model given as:

Model 1:

$$\max \quad \frac{\sum_{r=1}^{q} \mu_{rk} y_{rk}}{\sum_{i=1}^{p} \vartheta_{ik} x_{ik}}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{q} \mu_{rk} y_{rj}}{\sum_{i=1}^{p} \vartheta_{ik} x_{ij}} \leq 1 \qquad \text{for } j = 1,2,..,n$$

$$\mu_{rk} \geq 0 \qquad \text{for } r = 1,2,..,q$$

$$\vartheta_{ik} \geq 0 \qquad \text{for } i = 1,2,..,p$$

Let the solution of above is μ_{rk}^* and ϑ_{ik}^* and self-efficiency score is θ_{kk}^*.

Next, cross efficiency of DMU k with respect to any other DMU j is obtained by using the optimal weights of DMU j which are obtained by the model given as:

Model 2:

$$\max \quad \frac{\sum_{r=1}^{q} \mu_{rj} y_{rj}}{\sum_{i=1}^{p} \vartheta_{ij} x_{ij}}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{q} \mu_{rj} y_{rt}}{\sum_{i=1}^{p} \vartheta_{ij} x_{it}} \leq 1 \qquad \text{for } t = 1,2,..,n$$

$$\mu_{rj} \geq 0 \qquad \text{for } r = 1,2,..,q$$

$$\vartheta_{ij} \geq 0 \qquad \text{for } i = 1,2,..,p$$

Let the solution of above is μ_{rj}^* and ϑ_{ij}^*. The cross-efficiency of DMU k is then given by

$$\theta_{kj}^* = \frac{\sum_{r=1}^{q} \mu_{rj}^* y_{rk}}{\sum_{i=1}^{p} \vartheta_{ij}^* x_{ik}}$$

If $\theta_{kj}^* > \theta_{kk}^*$ this implies $\frac{\sum_{r=1}^{q} \mu_{rj}^* y_{rk}}{\sum_{i=1}^{p} \vartheta_{ij}^* x_{ik}} > \frac{\sum_{r=1}^{q} \mu_{rk}^* y_{rk}}{\sum_{i=1}^{p} \vartheta_{ik}^* x_{ik}}$

Also μ_{rj}^* and ϑ_{ij}^* satisfy

$$\frac{\sum_{r=1}^{s} \mu_{rj} y_{rt}}{\sum_{i=1}^{m} \vartheta_{ij} x_{it}} \leq 1 \qquad \text{for } t = 1,2,..,n$$

$$\mu_{rj} \geq 0 \qquad \text{for } r = 1,2,..,q$$

$$\vartheta_{ij} \geq 0 \qquad \text{for } i = 1,2,..,p$$

This cannot be true as the solution as μ_{rk}^* and ϑ_{ik}^* are optimal.

This implies $\theta_{kj}^* \leq \theta_{kk}^*$

The above is true for all DMU $j = 1,2, \dots \dots n$

Therefore, we get $\theta_{kj}^* \leq \theta_{kk}^* \qquad for\ j = 1,2 \dots \dots n$ ■

THEOREM 2. *All cross efficiencies satisfy the basic definition of efficiency i.e., all cross efficiencies for a DMU are greater than or equal to zero and less than or equal to one.*

Mathematically, for a DMU k,

$$1 \geq \theta_{kj} \geq 0 \quad for\ all\ j$$

PROOF. Since all the inputs and outputs are real values, so y_{rk} and $x_{ik} \geq 0 \ \forall\ r, i$. Also the choice of weights μ_{rj} and ϑ_{ij} for any DMU $j = 1,2, \dots n$, is greater than equal to zero with at least one of the ϑ_{ij} non zero. Therefore, from definition of cross efficiency of DMU k, we have

$$\theta_{kj} = \frac{\sum_{r=1}^{q} \mu_{rj} y_{rk}}{\sum_{i=1}^{p} \vartheta_{ij} x_{ik}} \geq 0$$

Also, the constraint k in model 2 is

$$\frac{\sum_{r=1}^{q} \mu_{rj} y_{rk}}{\sum_{i=1}^{p} \vartheta_{ij} x_{ik}} \leq 1$$

Which implies $\theta_{kj} = \frac{\sum_{r=1}^{q} \mu_{rj} y_{rk}}{\sum_{i=1}^{p} \vartheta_{ij} x_{ik}} \leq 1$

Combining the above two, we get $0 \leq \theta_{kj} \leq 1$.

The above is true for all DMUs. This proves Theorem 2. ■

THEOREM 3. $0 \leq \max_{j \epsilon N}\{\theta_{jk}\} \leq \theta_{kk} \leq 1$,

where N is the index set for DMUs.

PROOF. Clear from Theorem 1, Theorem 2 and basic definition of self-efficiency. ■

THEOREM 4. *The benevolent cross efficiency is greater than or equal to aggressive cross efficiency for a DMU.*

PROOF. The benevolent and aggressive cross efficiencies are given by models as follows:

The benevolent cross efficiency

Model 3:

$$\text{Maximize} \quad \sum_{r=1}^{q} u_{rk} \left(\left(\sum_{j=1}^{n} y_{rj}\right) j \neq k\right)$$

$$\text{such that} \quad \sum_{i=1}^{p} v_{ik} x_{ik} = 1$$

$$\sum_{r=1}^{q} u_{rk} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{p} v_{ik} x_{ik} \leq 0$$

$$\sum_{r=1}^{q} u_{rk} y_{rj} - \sum_{i=1}^{p} v_{ik} x_{ij} \leq 0 \qquad for\ j = 1,2,..,n\ ,j \neq k$$

$$u_{rk} \geq 0 \qquad for\ r = 1,2,..,q$$

$$v_{ik} \geq 0 \qquad for\ r = 1,2,..,p$$

And aggressive cross efficiency

Model 4:

$$\text{Minimize} \quad \sum_{r=1}^{q} u_{rk} \left(\left(\sum_{j=1}^{n} y_{rj}\right) j \neq k\right)$$

$$\text{such that} \quad \sum_{i=1}^{p} v_{ik} x_{ik} = 1$$

$$\sum_{r=1}^{q} u_{rk} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{p} v_{ik} x_{ik} \leq 0$$

$$\sum_{r=1}^{q} u_{rk} y_{rj} - \sum_{i=1}^{p} v_{ik} x_{ij} \leq 0 \qquad for\ j = 1,2,..,n\ ,j \neq k$$

$$u_{rk} \geq 0 \qquad for\ r = 1,2,..,q$$

$$v_{ik} \geq 0 \qquad for\ r = 1,2,..,p$$

Where θ_{kk}^{*} is the self-efficiency for DMU k.

Observe model 3 and model 4 for their objective functions which are given as

$\max \sum_{r=1}^{q} u_{rk} \left(\left(\sum_{j=1}^{n} y_{rj}\right) j \neq k\right)$ and

$\min \sum_{r=1}^{q} u_{rk} \left(\left(\sum_{j=1}^{n} y_{rj}\right) j \neq k\right)$

Under the same set of constraints.

As Maximization is greater than equal to minimization of any function under the same set of constraints therefore

$\max \sum_{r=1}^{q} u_{rk} \left(\left(\sum_{j=1}^{n} y_{rj}\right) j \neq k\right) \geq$

$\min \sum_{r=1}^{q} u_{rk} \left(\left(\sum_{j=1}^{n} y_{rj}\right) j \neq k\right)$

This implies that

Benevolent cross-efficiency ≥ Aggressive cross efficiency

For a DMU.

This proves Theorem 4. ■

In the next section, the results of above theorems are verified by considering a numerical example.

V. A NUMERICAL EXAMPLE

To illustrate the above theorems, 14 DMUs set is considered each having three inputs and four outputs from a real data set of Sherman and Gold (1985) bank branches.

Details of Inputs are:

Input 1(I1): Rent (thousands of dollars).

Input 2(I2): Full time equivalent personnel.

Input 3(I3): Supplies (thousands of dollars).

Details of Outputs are:

Output 1(O1): Loan applications, new pass-book loans, life insurance sales.

Output 2(O2): New accounts, closed accounts

Output 3(O3): Travelers checks sold, bonds sold, bonds redeemed.

Output 4(O4): Deposits, withdrawals, checks sold, treasury checks issued, B% checks, loan payments, pass-book loan payments, life insurance payments, mortgage payments.

The numerical values of each of the inputs and outputs for 14 DMUs are given in table 1.

Table 1: Input and Output values for 14 DMUs

DMU	I1	I2	I3	O1	O2	O3	O4
D1	140000	42900	87500	484000	4139100	59860	2951430
D2	48800	17400	37900	384000	1685500	139780	3336860
D3	36600	14200	29800	209000	1058900	65720	3570050
D4	47100	9300	26800	157000	879400	27340	2081350

D5	32600	4600	19600	46000	370900	18920	1069100
D6	50800	8300	18900	272000	667400	34750	2660040
D7	40800	7500	20400	53000	465700	20240	1800250
D8	31900	9200	21400	250000	642700	43280	2296740
D9	36400	76000	21000	407000	647700	32360	1981930
D10	25700	7900	19000	72000	402500	19930	2284910
D11	44500	8700	21700	105000	482400	49320	2245160
D12	42300	8900	25800	94000	511000	26950	2303000
D13	40600	5500	19400	84000	287400	34940	1141750
D14	76100	11900	32800	199000	694600	67160	3338390

To verify the statements of theorems, firstly basic CCR models for 14 DMUs are formulated and solved to get their self-efficiency scores and weights. Then, these weights are used to find cross-efficiency scores for the rest of the DMUs. These cross-efficiency scores so obtained are tabulated in table 2 as cross efficiency matrix.

Table 2 lists all the cross efficiencies scores for 14 DMUs. The principal diagonal elements of table 2 represent self-efficiency scores. When we look column-wise, entries represent cross efficiency for a DMU mentioned in the heading of that column. For example, in column 1, the first entry represents the self-efficiency of DMU1, the second entry represents the cross efficiency of DMU1 with respect to weights of DMU2, and so on, the last entry represents the cross-efficiency of DMU1 with respect to weights of DMU14. From entries of table 2, one may observe that each entry in any row is greater than zero and less than or equal to one. Thus, from table 2, the statement of Theorem 1 is verified.

Table 2: Cross-Efficiency Matrix

DMU	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**
D1	1	0.94	0.751	0.694	0.4	0.746	0.483	0.635	0.652	0.448	0.47	0.419	0.313	0.448
D2	0.149	1	0.627	0.203	0.203	0.239	0.174	0.474	0.304	0.271	0.388	0.223	0.301	0.309
D3	0.216	0.701	1	0.453	0.336	0.537	0.452	0.738	0.558	0.911	0.517	0.558	0.288	0.45
D4	0.837	0.981	0.875	1	0.9	1	0.764	0.836	0.097	0.733	0.732	0.748	0.649	0.781
D5	0.839	1	0.879	1	0.904	1	0.762	0.841	0.097	0.728	0.74	0.747	0.661	0.788
D6	0.215	0.598	0.784	0.698	0.725	1	0.749	0.779	0.081	0.902	0.805	0.807	0.648	0.875
D7	0.722	1	1	0.944	0.808	1	0.782	0.924	0.122	0.902	0.791	0.817	0.615	0.802
D8	0.466	1	0.666	0.607	0.392	1	0.302	1	0.22	0.39	0.509	0.415	0.541	0.595
D9	0.309	0.704	0.511	0.298	0.126	0.479	0.116	0.701	1	0.251	0.211	0.199	0.185	0.234
D10	0.255	0.73	1	0.601	0.456	0.858	0.639	0.856	0.298	1	0.731	0.712	0.432	0.688
D11	0.256	1	0.914	0.709	0.785	1	0.727	0.904	0.094	0.853	0.967	0.799	0.869	1
D12	0.268	0.929	1	0.736	0.727	1	0.76	0.948	0.116	1	0.926	0.852	0.723	0.928
D13	0.246	1	0.864	0.679	0.786	0.971	0.694	0.866	0.085	0.781	0.956	0.757	0.905	1
D14	0.246	0.947	0.869	0.699	0.793	1	0.721	0.87	0.087	0.823	0.95	0.785	0.877	1

We also observe that for each row (and column) diagonal entry is maximum within each row (or column) which establishes that the cross efficiencies for a DMU are less than or at most equal to its self-efficiency. This verifies the statement of Theorem 2.

Next, the best cross efficiency for each DMU is calculated by taking maximum over its all cross efficiencies for each DMU k=1, 2,…,14. Self-Efficiency(SE) and best cross efficiencies(BCE) are then put together as shown in table 3.

Table 3: Self Efficiency (SE) and Best Cross Efficiency (BCE)

DMU	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**
SE	1	1	1	1	0.904	1	0.782
BCE	0.839	1	1	1	0.9	1	0.76
DMU	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**
SE	1	1	1	0.967	0.852	0.905	1
BCE	0.948	0.652	0.911	0.956	0.817	0.877	0.928

One may clearly observe from table 3 that SE $\geq$ BCE for all DMUs. This verifies the statement of Theorem 3.

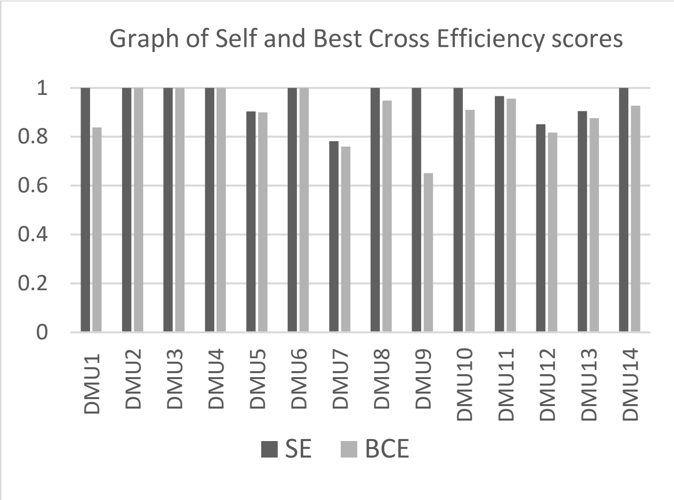

Fig. 1: Self and cross efficiency scores for 14 DMUs.

Figure 1 depicts the comparative bar graph for SE and BCE for 14 DMUs. The figure also substantiates the statement of Theorem 3, as SE bars are higher than BCE bars for DMUs. Similarly, the statement of Theorem 4 can be verified.

VI. Conclusion

In the present work, the relationship between self and cross efficiencies are examined and established. Mathematical formulations are developed in the form of four theorems and theoretical proofs are provided to establish the statements. The results are verified through numerical example and graphs. These relationships would develop a better understanding of these scores and would also be useful in DEA's further formulations like goal programming models.

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