Mean Cordial Labeling, Friendly labeling and Zero-edge magic labeling of Corona Graph $C_n o K_1$

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Abstract: Graph Theory is one of the branches of Mathematics. Graphs considered in Graph theory are discrete structures consisting of points and lines which connect these points. Graph labeling is assignment of numbers to the points and lines. Points are called vertices and lines are called edges. The graphs labeled using the vertices or edges of the graph is called labeled graph. Labeled graphs are used as models in different areas. One of the type of graph is Corona Graph $C_n o K_1$ which has cycle $C_n$ and n-copies of $K_1$ is attached to each vertex of $C_n$. In this paper, Mean Cordial labeling, Friendly labeling and Zero-edge magic labeling of Corona Graph will be discussed.

Index Terms: Corona Graph $C_n o K_1$, Friendly labeling, Mean Cordial labeling, Zero-edge magic labeling.

I. INTRODUCTION

In a graph G, an assignment of integers to the vertices or edges or both using certain conditions is called labeling of graph. Most of the terminologies and notations are taken from Harary [3]. Some survey of graph labelings is done from Gallian [2]. Let $V(G)$ denote the vertex set and $E(G)$ denote the set of edges. The graph G is denoted as ordered pair of $V(G)$ and $E(G)$ i.e. $G = (V(G), E(G))$. There are many types of labelings techniques. Some of them are E-cordial labeling, prime labeling, harmonic labeling, magic labeling etc. applied to certain classes of graphs. Some of them are vertex labelings and edge labelings are induced from it while some are edge labelings and vertex labelings are induced from it. Corona graphs $C_n o K_1$ are simple and undirected graphs. In this paper, mean cordial labeling, friendly labeling and zero-edge magic labeling of Corona graphs $C_n o K_1$ are discussed. The labelings are vertex labelings and edge labelings are induced from it. These labelings can be given to many classes of graph.

II. PRELIMINARIES AND NOTATION

Here, in this part, the basic definitions are given for development of paper.

Definition 2.1 [3]: Cycle graph $C_n$: A connected graph with n edges in which initial and final vertex are same.

Definition 2.2: Corona graph $C_n o K_1$[3]: The corona $G_1 o G_2$ of two graphs $G_1$ and $G_2$ is a graph G obtained by taking one copy of $G_1$ which has $p_1$-vertices and $p_1$-copies of $G_2$ and then joining $i^{th}$ vertex of $G_1$ to every vertex in the $i^{th}$ copy of $G_2$.

III. MAIN RESULTS

3.1. Mean Cordial Labeling of corona graph $C_n o K_1$.

Definition 3.1.1: Mean Cordial Labeling [7]: For each edge uv of graph G assign the label $f^*(uv) = \lceil \frac{f(u) + f(v)}{2} \rceil$ (ceiling function). Then the map $f : V(G) \to \{0, 1, 2\}$ is called mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$; i. j $\in \{0, 1, 2\}$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges respectively labeled with x.

Definition 3.1.2: Mean Cordial Graph [7]: A graph G which admits mean cordial labeling is called mean cordial graph.

Theorem 3.1.3: The corona graph $C_n o K_1$ is mean cordial graph for $n \equiv 2 (mod 3)$ and $n \geq 3$. 

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Proof: Let \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\} be the set of vertices of \(G = C_n o K_1\).

The vertices on cycle \(C_n\) are \(u_1, u_2, \ldots, u_n\) while \(v_1, v_2, \ldots, v_n\) are pendant vertices adjacent to \(u_1, u_2, \ldots, u_n\) respectively.

Define labeling function \(f: V(G) \to \{0, 1, 2\}\) of vertices of \(G = C_n o K_1\) as follows.

\[
\begin{align*}
    f(u_i) &= 0 \quad \text{for } 1 \leq i \leq \frac{n+1}{3} \quad \text{[cycle vertices]} \\
    &= 2 \quad \text{for } \frac{n+4}{3} \leq i \leq \frac{2n+2}{3} \\
    &= 1 \quad \text{for } \frac{2n+5}{3} \leq i \leq n-1.
\end{align*}
\]

\[
\begin{align*}
    f(v_i) &= 0 \quad \text{for } 1 \leq i \leq \frac{n+1}{3} \quad \text{[pendant vertices]} \\
    &= 2 \quad \text{for } \frac{n+4}{3} \leq i \leq \frac{2n-1}{3} \\
    &= 1 \quad \text{for } \frac{2n+2}{3} \leq i \leq n.
\end{align*}
\]

This labeling of vertices induces the edge labelings as follows:

\[
\begin{align*}
    f(e_i) = f(u_i, u_{i+1}) &= 0 \quad \text{for } 1 \leq i \leq \frac{n-2}{3} \quad \text{[cycle edges]} \\
    &= 2 \quad \text{for } \frac{n+4}{3} \leq i \leq \frac{2n+2}{3} \\
    &= 1 \quad \text{for } \frac{2n+5}{3} \leq i \leq n-1.
\end{align*}
\]

\[
\begin{align*}
    f(e_i) = f(u_i, v_i) &= 1 \quad \text{if } i = \frac{n+1}{3}, \\
    f(e_i) = f(v_i, v_{i+1}) &= 1 \quad \text{if } i = \frac{n-1}{3}, \\
    f(e_i) = f(v_{i+1}, u_i) &= 1 \quad \text{for } \frac{2n+2}{3} \leq i \leq n.
\end{align*}
\]

This vertex labeling and induced edge labels shows that

\[
\begin{align*}
    v_i(0) &= \frac{2n+2}{3}, \quad v_i(1) = \frac{2n-1}{3}, \\
    v_i(2) &= \frac{2n-1}{3} \quad \text{and} \\
    e_{r}(0) &= \frac{2n-1}{3}, \quad e_{r}(1) = \frac{2n-1}{3}, \\
    e_{r}(2) &= \frac{2n+2}{3}
\end{align*}
\]

which satisfies \(|v_r(i) - v_r(j)| \leq 1\) and \(|e_r(i) - e_r(j)| \leq 1\) for all \(i, j \in \{0, 1, 2\}\).

Hence, corona graph \(C_n o K_1\) is mean cordial graph for \(n \equiv 2 \pmod{3}\) and \(n \geq 3\).

Remark 3.1.4: When \(n \equiv 0 \pmod{3}\) and \(n \equiv 1 \pmod{3}\) the graph cannot be mean cordial graph.

Case(i) When \(n \equiv 0 \pmod{3}\), then \(|V(C_n o K_1)| = 2n = |E(C_n o K_1)|\).

Also, \(v_r(i) = \frac{2n}{3}\), \(v_r(1) = \frac{2n-3}{3}\), \(v_r(2) = \frac{2n-1}{3}\), \(e_{r}(0) = \frac{2n}{3}\), \(e_{r}(1) = \frac{2n+3}{3}\), and \(e_{r}(2) = \frac{2n+3}{3}\).

Hence, \(|e_r(i) - e_r(j)| \leq 1\) is not satisfied for all \(i, j \in \{0, 1, 2\}\).

Case(ii) When \(n \equiv 1 \pmod{3}\), then \(|V(C_n o K_1)| = 2n - 3|E(C_n o K_1)|\).

Also, \(v_r(i) = \frac{2n-2}{3}, v_r(1) = \frac{2n+1}{3}, v_r(2) = \frac{2n+1}{3}\) while \(e_{r}(0) = \frac{2n-5}{3}, e_{r}(1) = \frac{2n+1}{3}, e_{r}(2) = \frac{2n+4}{3}\).

Hence, \(|e_r(i) - e_r(j)| \leq 1\) is not satisfied for all \(i, j \in \{0, 1, 2\}\).

3.2. Friendly labeling of corona graph \(C_n o K_1\).

Notations and terminologies in 3.2 are taken from[1].

Definition 3.2.1: Friendly labeling.

Let \(G\) be a graph and \(f: V(G) \to \mathbb{Z}_2\) be a binary vertex labeling of \(G\). For \(i \in \mathbb{Z}_2\) let, \(v_i = |f(i)|\). Then the labeling \(f\) is said to be friendly if \(|v_0 - v_1| = 1\).

Definition 3.2.2: Product - cordial index or pc-index.

Any friendly labeling \(f: V(G) \to \mathbb{Z}_2\) induces an edge labeling \(f^*: E(G) \to \mathbb{Z}_2\) defined by \(f^*(xy) = f(x)f(y) \neq xy \in E(G)\).

For \(i \in \mathbb{Z}_2\), let \(e_r(i) = \left|\left|f^*(i)\right|-\left|f^*(i)\right|\right|\) be the number of edges of \(G\) that are labeled \(i\). The number \(pc(f) = |e_r(1) - e_r(0)|\) is called the product-cordial index or pc-index of \(f\).

Definition 3.2.3: Product-Cordial set or pc-set.

The product cordial set or pc-set of the graph \(G\), denoted by \(PC(G)\), is defined by \(PC(G) = \{pc(f) : f \text{ is friendly vertex labeling of } G\}\).

Definition 3.2.4: Product-cordial graph.

A graph with friendly labeling is called product cordial graph.

Definition 3.2.5: Fully product-cordial or fully pc graph.

A graph \(G\) of size \(q\) is said to be fully product cordial or fully pc if \(PC(G) = \{q - 2k : 0 \leq k \leq \left\lfloor q / 2 \right\rfloor\}\) where \(\left\lfloor x \right\rfloor\) denotes greatest integer lesser than or equal to \(x\).

Example 3.2.6: Friendly labeling of a graph.

Figure 2: Friendly labeling of graph \(G\)

Consider the graph \(G\) of figure 2 which has six vertices. The condition

\(|v_0(1) - v_0(0)| \leq 1\) implies that three vertices are labeled 0 and the other three 1. Product cordial index for this labeling is \(pc(f) = |e_r(1) - e_r(0)| = |9 - 1| = 8\).
Therefore, $PC(G) = \{ pc(f) : f \text{ is friendly vertex labeling of } G\} = \{4, 6, 8\}$.

This friendly labeling does not give fully product cordial graph.

**Theorem 3.2.7:** For any $n \geq 3$, the corona graph $C_n \circ K_1$ have friendly-labeling and is fully product cordial. Also, $PC(C_n \circ K_1) = \{2(n-k) : 0 \leq k \leq n\}$

**Proof:** Consider corona graph $C_n \circ K_1$ with vertex set $V$ and edge set $E$. $|V| = 2n = |E|$

Let $\{u_1, u_2, \ldots, u_n\}$ be the vertices on the cycle $C_n$. Let $\{v_1, v_2, \ldots, v_n\}$ be the vertices pendant to $u_1, u_2, \ldots, u_n$ respectively. The labeling of corona graph can be done as follows:

1) For $pc(f) = 0$, label all vertices on cycle as 1 and all pendant vertices as zero. Here $v_f(0) = n = v_f(1)$ and $e_f(0) = n = e_f(1)$

2) For $pc(f) = 2n$, label all vertices on cycle as zero and all pendant vertices as 1. Here $v_f(0) = n = v_f(1)$ and $e_f(0) = 2n$, $e_f(1) = 0$.

3) For $pc(f) = 2, 4, 6, 8, \ldots, 2n-2$ and for $i = 1, 2, \ldots, n-1$

   Label ‘$n-i$’ consecutive vertices on cycle by 1 and remaining ‘$i$’ vertices by zero. Label one pendant vertex incident to 1 by 1 and all the remaining vertices pendant to label 1 will be labeled 0. Label ‘$i-1$’ pendant vertices incident to vertex with label zero by 1 and remaining all pendant vertices incident to vertex with label zero by 0. Here for $1 \leq i \leq n-1$, $v_f(0) = n = v_f(1)$, $e_f(0) = n + i$ and $e_f(1) = n - i$. Therefore $pc(f) = 2i$. The above labeling makes $C_n \circ K_1$ fully product-cordial.

**Illustration 3.2.8:** Consider $C_5 \circ K_1$.

Here, $n = 5$.

1) $v_f(0) = v_f(1) = 5$
   $e_f(0) = 5 = e_f(1)$
   $\therefore pc(f) = 0.$

2) $v_f(0) = 5 = v_f(1)$
   $e_f(0) = 4$, $e_f(1) = 6$
   $\therefore pc(f) = 2.$

3) $v_f(0) = 5 = v_f(1)$
   $e_f(0) = 3$, $e_f(1) = 7$
   $\therefore pc(f) = 4.$

4) $v_f(0) = v_f(1) = 5$
   $e_f(0) = 2$, $e_f(1) = 8$
   $\therefore pc(f) = 6.$

5) $v_f(0) = v_f(1) = 5$
   $e_f(0) = 1$, $e_f(1) = 9$
   $\therefore pc(f) = 8.$
6) \( v_1(0) = v_2(1) = 5 \)
\( e_1(0) = e_2(1) = 0 \)
\[ \therefore \text{pc}(f) = 10. \]
Thus, PC(C\(^n\times K_1\)) = \{pc (f) : f is friendly labeling of C\(^n\times K_1\}\} = \{0, 2, 4, 6, 8, 10\}
= \{q – 2k : 0 \leq k \leq \lfloor q/2 \rfloor \} \text{ where } q = 10.
Hence, C\(^n\times K_1\) is fully product cordial graph.

### 3.3. Zero-edge magic labeling of corona graph C\(^n\times K_1\).

Notations and terminologies in 3.3 are taken from[4].

**Definition 3.3.1:** Zero-edge magic labeling.
Let \( G=(V, E) \) be a graph. Let \( f : V \rightarrow \{-1, 1\} \) and \( f^* : E \rightarrow \{0\} \) such that all \( uv \in E, f^*(uv)=f(u) + f(v) = 0 \) then the labeling is said to be zero-edge magic labeling of graph \( G \).

**Definition 3.3.2:** Zero-edge magic graph.
A graph which admits zero edge magic labeling is called zero-edge magic graph.

**Theorem 3.3.3:** The corona graph C\(^n\times K_1\) is zero edge magic for \( n \) even.

**Proof:** Let \( u_1, u_2, \ldots, u_n \) be vertices of \( C_n \) and \( v_1, v_2, \ldots, v_n \) be the vertices pendant to \( u_1, u_2, \ldots, u_n \) respectively. Define labeling \( f : V \rightarrow \{-1, 1\} \) and \( f^* : E \rightarrow \{0\} \) as follows:
\[
f(u_i) = (-1)^i \quad 1 \leq i \leq n
\]
\[
f(v_i) = (-1)^{i+1} \quad 1 \leq i \leq n
\]
Then,
\[
f^*(u_iu_{i+1}) = f(u_i) + f(u_{i+1}) = (-1)^i + (-1)^{i+1} = -1 + 1 = 0
\]
if \( i \) is odd, \( 1 \leq i \leq n-1 \)
\[
f(u_iu_i) = f(u_i) + f(u_i) = 1 + (-1) = 0
\]
if \( i \) is even, \( 1 \leq i \leq n-1 \)
\[f(u_iu_1) = 0.\]
And \( f^*(u_iv_i) = f(u_i) + f(v_i) = (-1)^i + (-1)^{i+1} = -1 + 1 = 0\)
if \( i \) is odd
\[
if \( i \) is even

Hence, C\(^n\times K_1\) admits zero edge magic labeling and C\(^n\times K_1\) is zero edge magic graph.

**Illustration 3.3.4:** Consider C\(_6\times K_1\).
Here \( n = 6 \).

\[
f(u_i) = (-1)^i; \quad 1 \leq i \leq 6
f(v_i) = (-1)^{i+1}; \quad 1 \leq i \leq 6 \]
Then \( \therefore \text{C}_{6}\times K_1 \) is zero-edge magic graph.

**Remark 3.3.5:** When \( n \) is odd, one of the induced edge label is either 2 or -2, hence the graph is not zero-edge magic graph.

**CONCLUSION AND FUTURE RESEARCH DIRECTIONS**
A particular class of graph admit different labelings techniques. Similar results can be obtained in the context of different graphs. Algorithms can be developed for these labeling. Applications of these labelings with covering problem to project Management.

**REFERENCES**


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