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Difference Squeezing in Degenerate Three-Photon Absorption Six-Wave Interaction Process

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Abstract: In this paper, difference squeezing in the degenerate three-photon absorption six-wave interaction process is examined. The difference squeezing in fields between the pump and Stokes modes can be converted to normal squeezing in the signal mode, and vice versa. It is demonstrated that in the pump mode, amplitude-cubed squeezing is directly turned into normal squeezing in the signal mode. The squeezing in the stimulated interaction, which reduces the depth of classicality of the field amplitude, is seen to be larger than the corresponding squeezing in the spontaneous interaction, despite having the same number of pump photons. Difference squeezing is observable only in certain domain values of pump photons. It is deduced that the multi-photon absorption nonlinear optical approach is the best way to generate optimal squeezed laser light in any optical system.

Keywords: Amplitude-cubed Squeezing, Difference Squeezing, Photon Number Operator, Six-Wave Interaction Process, Squeezed States.

I. INTRODUCTION

Squeezed states of light (Walls, 1983; Loudon & Knight, 1987; Henry & Glotzer, 1988; Teich & Saleh, 1989) are one of the intrinsic examples of nonclassical states of light (Mandel, 1986; Dodonov, 2002; Wódkiewicz, 1987). It may be expressed using complex amplitude, which represents both the amplitude and phase of the field. The amplitude of the electromagnetic field is never constant; there are always quantum residual fluctuations (Slusher et. al., 1985; Bachor, 1998; Vogel et.al., 2001), called zero-point fluctuations. The fluctuations in the quadrature components are equal and randomly distributed in the field quadrature components in a coherent condition (Perina, 1991). On the other hand, it is feasible to minimize fluctuations in one quadrature component while increasing fluctuations in the other quadrature component when compared to a coherent state. These states are called squeezed states (Loudon, 2000; Dodonov, 2007), in which quantum noise (or fluctuations) in field quadrature components is not randomly distributed. Quantum low-noise exists in all electromagnetic field states, including the field vacuum state. Its low-noise fluctuation in any quantum state (Special issue on squeezed states, 1987, p. 707; 1987, p. 1450) has piqued the interest of the community, with potential applications in optical telecommunication (Saleh & Teich, 1987*;* Yuen & Shapiro, 1978), quantum cryptography (Bennett et. al., 1992; Kempe, 1999), an interferometric approach to detect gravitational waves (Caves, 1981), signal amplification (Wong, 1991), and so on. Squeezed states are a novel sort of quantum state in the electromagnetic field, and study into them should yield new basic insights. Further, the concept of higher-order squeezing of the quantized electromagnetic field was introduced by Hong et al. (Hong & Mandel, 1985, p. 323; 1985, p. 974) and Hillery (Hillery, 1987, p. 3796; 1987, p. 135). Based on theoretical and experimental evidence, several researchers have reported the development of techniques for measuring higher-order correlations in quantum optics in various nonlinear optical processes such as parametric amplification (Wu et. al., 1986; Fernee et. al., 1995), harmonic generations (Mandel, 1982; Kielich et. al., 1987; Zhan, 1991; Pratap et. al., 2014,

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p.1065; Pratap et. al. 2014, p. 1126), multi-photon processes (Reid & Walls, 1984, p. 406; 1985, p. 1622; Perina et. al., 1984; Razmi & Eberly, 1990; Tanas et. al., 1991; Hillery, 1992; Giri & Gupta, 2003, p. 135; 2008, 219; Choudhary & Giri, 2018), and Raman processes (Perina & Krepelka, 1991; Kumar & Gupta, 1995, p. 835; 1996, p. 1053; Giri & Gupta, 2005). In their recent research, Prakash and Mishra have recently looked into higher-order squeezing as a way to improve the performance of a number of optical devices and networks (Prakash & Mishra, 2005, p. 665; 2010, p. 2212; Mishra, 2010). Garcia Fernandez et al. (Garcia et. al., 1986) as well as Mishra et al. (Prakash & Mishra, 2006; Mishra & Singh, 2020) investigated higher-order nonclassical states in single-mode and their utility in detecting nonclassical light. Hillery (Hillery, 1989) proposed a two-mode sum and difference squeezing that was later expanded to three modes (Kumar & Gupta, 1998) and an arbitrary number of modes (Giri & Gupta, 2005; Olsen & Horowicz, 2007; Ba & Tinh, 2000). Prakash et al. (Prakash & Mishra, 2007) investigated the production of sum squeezing in two-mode light when mixed with coherent light using a beam splitter. Truong et al. (Truong et. al., 2014) and Wang et al. (Wang & Xu, 2015) have investigated the precise behaviour of higher-order nonclassical effects and entanglements as a function of the parameters involved. Mukherjee et al. (Mukherjee et. al., 2016) discussed the possibilities of sum-and-difference squeezing in harmonic generating techniques. Giri et al. (Giri & Choudhary, 2020) investigated the possibility of generating sum squeezing in the frequency up-conversion process, and more recently, Mishra et al. (Mishra et. al., 2020) described how a beam splitter with third-order nonlinear material could generate sum-and difference-squeezing that is relevant for applications to efficient quantum computation, quantum teleportation, and other quantum communication and information problems. As a result, it opens up new possibilities for exploring higher order nonclassical effects.

In the present paper, we examine the squeezing of differences between the pump and Stokes modes in a degenerate three-photon absorption six-wave interaction process, in which three pump photons interact with a nonlinear medium, resulting in the emission of two probe (Stokes) photons and the subsequent emission of one signal photon to the initial state. The format of the paper is as follows: Higher-order squeezing is defined in Section II. Section III establishes the analytic equation for difference squeezing in the six-wave difference-frequency of the two modes. The results and discussion are presented in Section IV. Finally, the paper is summarized and concluded in section V.

II. DEFINITION OF HIGHER-ORDER SQUEEZING

Higher-order squeezing is the higher powers of the field amplitude (Hillery, 1987), which is illustrated in the following two sections.

A. AMPLITUDE-CUBED SQUEEZING OF SINGLE MODE

The real and imaginary portions with frequency ω and creation (annihilation) operators $\hat{a}^{\dagger}(\hat{a})$ of the amplitude-cubed squeezing (Zhan, 1991) for a single mode of an optical field can be defined as follows:

$$\hat{Z}_1 = (1/2)(\hat{A}^3 + \hat{A}^{\dagger 3}) \quad (1)$$

And

$$\hat{Z}_2 = (1/2i)(\hat{A}^3 - \hat{A}^{\dagger 3}) \quad (2)$$

where $\hat{A}$ and $\hat{A}^{\dagger}$ are the gradually varying operators, they are given by

$$\hat{A} = \hat{a}\ \exp(i\omega t) \text{ and } \hat{A}^{\dagger} = \hat{a}^{\dagger}\ \exp(-i\omega t) \quad (3)$$

The commutation relation is followed by equations (1) and (2),

$$\left[\hat{Z}_1, \hat{Z}_2\right] = \frac{i}{2}\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right) \quad (4)$$

where $\hat{A}^{\dagger}\hat{A} = \hat{N}_{\hat{A}}$ is the number operator.

Equation (4) yields the uncertainty relation (ħ = 1), as

$$\Delta\hat{Z}_1 \Delta\hat{Z}_2 \geq \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle \quad (5)$$

where $\Delta\hat{Z}_1$ and $\Delta\hat{Z}_2$ are the uncertainties in the quadrature.

Amplitude-cubed squeezed state in $\hat{Z}_i$ exists if

$$\left(\Delta\hat{Z}_i\right)^2 < \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle \text{ where i = 1 or 2.} \quad (6)$$

The nature of these states is completely quantum mechanical. Traditionally, the variance $\left(\Delta\hat{Z}_i\right)^2$ is expressed in the state's *P*-representation as

$$\left(\Delta\hat{Z}_i\right)^2 = \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle + \frac{1}{4}\int d^2\alpha P(\alpha) \times\left[\exp(-3i\omega t)\alpha^{*3} + \exp(3i\omega t)\alpha^3 - \left\langle \hat{A}^3 + \hat{A}^{\dagger 3}\right\rangle\right]^2 \quad (7)$$

where *P*(α) is the quasi-probability function for coherent states.

A classical state whose *P*-representation is non-negative definite, as in equation (7), satisfies the connection,

$$\left(\Delta\hat{Z}_i\right)^2 \geq \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle \qquad (8)$$

This means that a state satisfying equation (6) has non-classical characteristics.

A coherent state is one in which the field quadrature variances fulfil the following equation:

$$\left(\Delta\hat{Z}_i\right)^2 = \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle . \qquad (9)$$

B. DIFFERENCE SQUEEZING OF TWO MODES

Difference squeezing may be defined through variables $\hat{W}_1$ and $\hat{W}_2$ in two modes having frequency ω_1 and ω_2 with creation (annihilation) operators $\hat{a}^\dagger(\hat{a})$ and $\hat{b}^\dagger(\hat{b})$ respectively, as

$$\hat{W}_1 = \left(\frac{1}{2}\right)\left(\hat{A}^3\hat{B}^{\dagger 2} + \hat{A}^{\dagger 3}\hat{B}^2\right) \qquad (10)$$

and $$\hat{W}_2 = \left(\frac{1}{2i}\right)\left(\hat{A}^3\hat{B}^{\dagger 2} - \hat{A}^{\dagger 3}\hat{B}^2\right) \qquad (11)$$

The operators $\hat{W}_1$ and $\hat{W}_2$ yield the commutation relation as

$$\left[\hat{W}_1, \hat{W}_2\right] = \frac{i}{2}\left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2N_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right) \qquad (12)$$

and leads to uncertainty relation (ħ = 1)

$$\Delta\hat{W}_1\Delta\hat{W}_2 \geq \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2N_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle . \qquad (13)$$

where $\hat{N}_{\hat{A}} = \hat{A}^\dagger\hat{A}$ and $\hat{N}_{\hat{B}} = \hat{B}^\dagger\hat{B}$ are the photon number operators.

Difference squeezing in $\hat{W}_j$ direction exists if

$$\left(\Delta\hat{W}_j\right)^2 < \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2N_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle$$

where j = 1 or 2 . (14)

For two-mode state, the P-representation P(α,β) may be expressed as,

$$\left(\Delta\hat{W}_j\right)^2 = \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2N_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle + \frac{1}{4}\int d^2\alpha\int d^2\beta P(\alpha,\beta)[\{\exp(2i\omega_2 t)\alpha^3\beta^{*2} \pm \exp(-2i\omega_2 t)\alpha^{*3}\beta^2\} - \langle\hat{W}_j\rangle]^2 . \qquad (15)$$

where j = 1 or 2 signifies for ± respectively.

III. DIFFERENCE SQUEEZING IN DEGENERATE THREE-PHOTON ABSORPTION SIX-WAVE INTERACTION PROCESS

The nonlinear interaction in this model (Ducloy, 1985), shown in Figure 1, occurs when three pump photons of frequency ω_1 each interact with a nonlinear medium, resulting in the emission of two probe (Stokes) photons of frequency ω_2 and the subsequent emission of one signal photon of frequency ω_3 to the initial state.

The theoretical predictions of the present study in six-wave interaction process can be experimentally verified and measured easily by using homodyne photon counting experiments (Bachor, 1998; Vogel et. al., 2001). In essence, the simple physical model (Ducloy, 1985) is experimentally attainable and readily visible in any nonlinear optics laboratory.

From figure 1, the corresponding interaction Hamiltonian is

$$\hat{H} = \omega_1\hat{a}^\dagger\hat{a} + \omega_2\hat{b}^\dagger\hat{b} + \omega_3\hat{c}^\dagger\hat{c} + g(\hat{a}^3\hat{b}^{\dagger 2}\hat{c}^\dagger + \hat{a}^{\dagger 3}\hat{b}^2\hat{c}) . \qquad (16)$$

where $\hat{a}^\dagger(\hat{a})$, $\hat{b}^\dagger(\hat{b})$ and $\hat{c}^\dagger(\hat{c})$ are the creation (annihilation) operators, g is the coupling interaction constant per second between the modes and slowly varying operators $\hat{A} = \hat{a}\exp(i\omega_1 t)$, $\hat{B} = \hat{b}\exp(i\omega_2 t)$ and $\hat{C} = \hat{c}\exp(i\omega_3 t)$ with the relation $3\omega_1 = 2\omega_2 + \omega_3$.

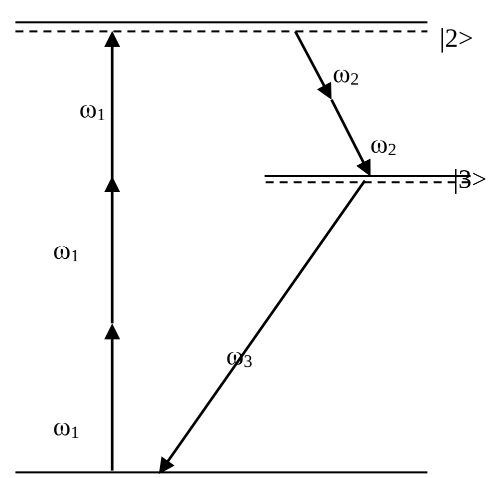

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Figure 1: Degenerate six-wave interaction model

Using equation (16) in Heisenberg equation of motion

$$\dot{\hat{A}} = \frac{\partial \hat{A}}{\partial t} + i\left[\hat{H}, \hat{A}\right] \quad .(\hbar = 1) \qquad (17)$$

we obtain

$$\dot{\hat{A}} = -3ig\hat{A}^{\dagger 2}\hat{B}^2\hat{C} \quad . \qquad (18)$$

Analogously, we arrive at

$$\dot{\hat{B}} = -2ig\hat{A}^3\hat{B}^{\dagger}\hat{C}^{\dagger} \quad . \qquad (19)$$

and
$$\dot{\hat{C}} = -ig\hat{A}^3\hat{B}^{\dagger 2} \qquad (20)$$

Using equations (18) and (19) equation (20) we obtain

$$\ddot{\hat{C}} = -|g|^2\left[9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right]\hat{C} \qquad (21)$$

The coupling constant (g) between the modes is employed $|g|^2$ instead of g^2 (Perina, 1991; Tanas et. al., 1991) in the interaction Hamiltonian.

Using Taylor's series up to second-order in '*gt*' (*gt* <<1) with short- time scale (≈10^{-10}sec) (Perina, 1991), we get

$$\hat{C}(t) = \hat{C}(0) + t\dot{\hat{C}}(0) + \left(t^2/2!\right)\ddot{\hat{C}}(0) + \ldots\ldots \qquad (22)$$

Inserting equations (20) and (21) in equation (22), gives

$$\hat{C}(t) = \hat{C} - igt\hat{A}^3\hat{B}^{\dagger 2} - \left(\frac{|g|^2 t^2}{2}\right) \times \left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right)\hat{C} \qquad (23)$$

and

$$C^{\dagger}(t) = C^{\dagger} + igt\hat{A}^{\dagger 3}\hat{B}^2 - \left(\frac{|g|^2 t^2}{2}\right) \times \left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right)\hat{C}^{\dagger} \quad . \qquad (24)$$

where the operators at $t = 0$ represents $\hat{C}(0) = \hat{C}$ throughout the paper.

We define two general quadrature components to investigate the squeezing in the signal mode

$$\hat{X}_{1\hat{C}}(t) = \left(\frac{1}{2}\right)[\hat{C}(t) + \hat{C}^{\dagger}(t)] \qquad (25)$$

and
$$\hat{X}_{2\hat{C}}(t) = \left(\frac{1}{2i}\right)[\hat{C}(t) - \hat{C}^{\dagger}(t)] \qquad (26)$$

Using equations (23) and (24) in equations (25) and (26) we obtain

$$\hat{X}_{1\hat{C}}(t) = \hat{X}_{1\hat{C}} + |g|t\left(\hat{W}_2\right) - \left(\frac{|g|^2 t^2}{2}\right) \times \left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right)\hat{X}_{1\hat{C}} \qquad (27)$$

and

$$\hat{X}_{2\hat{C}}(t) = \hat{X}_{2\hat{C}} - |g|t\left(\hat{W}_1\right) - \left(\frac{|g|^2 t^2}{2}\right) \times \left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right)\hat{X}_{2\hat{C}} \qquad (28)$$

Equations (27) and (28) becomes when at $t = 0$ the modes $\hat{A}$ and $\hat{B}$ are uncorrelated, as

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{1\hat{C}}\right)^2 + |g|^2 t^2\left(\Delta\hat{W}_2\right)^2 - |g|^2 t^2 \left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle \left(\Delta\hat{X}_{1\hat{C}}\right)^2 \qquad (29)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 + |g|^2 t^2\left(\Delta\hat{W}_1\right)^2 - |g|^2 t^2 \left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle \left(\Delta\hat{X}_{2\hat{C}}\right)^2 \qquad (30)$$

If the $\hat{C}$ mode is in a coherent state at the start, then

$$\left(\Delta\hat{X}_{1\hat{C}}\right)^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 = \left(\frac{1}{4}\right). \qquad (31)$$

Using equation (31) in equations (29) and (30), we have

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2[\left(\Delta\hat{W}_2\right)^2 - \left(\frac{1}{4}\right) \times \left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle] \qquad (32)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2[\left(\Delta\hat{W}_1\right)^2 - \left(\frac{1}{4}\right) \times \left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2 + 18\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^2 - 2\hat{N}_{\hat{A}}^3 + 6\hat{N}_{\hat{B}}^2 - 4\hat{N}_{\hat{A}}^3\hat{N}_{\hat{B}}\right\rangle]$$

(33)

Equations (32) and (33) show that difference-frequency generation will yield an output that is squeezed in the $\hat{X}_{1\hat{C}}$ or $\hat{X}_{2\hat{C}}$ direction if the input state is difference squeezing in the $\hat{W}_2$ or $\hat{W}_1$ direction respectively. As a result, difference squeezing in the pump and Stokes modes for the uncorrelated mode can be converted to normal squeezing in the signal mode and vice versa. This finding provides a link between difference and normal squeezing, as well as a way for detecting difference squeezing in the six-wave difference-frequency production process.

Furthermore, we assume that the change in the Stokes mode i.e. the $\hat{B}$-mode remains constant. Equation (20) is obtained by associating a constant term *m* for $\hat{B}$ and $\hat{B}^\dagger$ in the signal mode, then we have

$$\dot{\hat{C}} = -ig\hat{A}^3 m^2 \quad (34)$$

then $$\ddot{\hat{C}} = -|g|^2 m^4 \left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{C} \quad (35)$$

Hence, the corresponding results in the amplitude signal mode are

$$\hat{C}(t) = \hat{C} - i|g|t\hat{A}^3 m^2 - \left(\frac{|g|^2 t^2}{2}\right) m^4 \left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{C} \quad (36)$$

and

$$\hat{C}^\dagger(t) = \hat{C}^\dagger + i|g|t\hat{A}^{\dagger 3} m^2 - \left(\frac{|g|^2 t^2}{2}\right) m^4 \left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{C}^\dagger \quad (37)$$

Use of equations (36) and (37) in equations (25) and (26), we get

$$\hat{X}_{1\hat{C}}(t) = \hat{X}_{1\hat{C}} + |g|t\hat{Z}_{2\hat{A}} m^2 - \left(\frac{|g|^2 t^2}{2}\right) m^4 \left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{X}_{1\hat{C}} \quad (38)$$

and

$$\hat{X}_{2\hat{C}}(t) = \hat{X}_{2\hat{C}} - |g|tm\hat{Z}_{1\hat{A}} m^2 - \left(\frac{|g|^2 t^2}{2}\right) m^4 \left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{X}_{2\hat{C}} \quad (39)$$

where $\hat{Z}_{1\hat{A}}$ and $\hat{Z}_{2\hat{A}}$ define in equation (6).

Equations (38) and (39) becomes when the modes are uncorrelated at $t = 0$, then

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{1\hat{C}}\right)^2 + |g|^2 t^2 m^4 \times \left[\left(\Delta\hat{Z}_{2\hat{A}}\right)^2 - \left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\left(\Delta\hat{X}_{1\hat{C}}\right)^2\right] \quad (40)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 + |g|^2 t^2 m^4 \times \left[\left(\Delta\hat{Z}_{1\hat{A}}\right)^2 - \left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\left(\Delta\hat{X}_{2\hat{C}}\right)^2\right] \quad (41)$$

Using the condition of equation (31) in equations (40) and (41), then yields

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2 m^4 \times \left[\left(\Delta\hat{Z}_{2\hat{A}}\right)^2 - \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\right] \quad (42)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2 m^4 \times \left[\left(\Delta\hat{Z}_{1\hat{A}}\right)^2 - \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\right] \quad (43)$$

Equations (42) and (43) indicate that the signal mode is squeezed in the $\hat{X}_{1\hat{C}}$ direction if the pump mode is amplitude-cubed squeezed (third-order squeezing) in the $\hat{Z}_{2\hat{A}}$ direction, and the signal mode is squeezed in the $\hat{X}_{2\hat{C}}$ direction if the pump mode is amplitude-cubed squeezing in the $\hat{Z}_{1\hat{A}}$ direction. A squeezed signal mode (normal squeezing) is generated when a pump mode with amplitude-cubed squeezing (third-order squeezing) propagates across a nonlinear medium. The findings suggest a mechanism for detecting amplitude-cubed squeezing in degenerate six-wave interactions.

The following are the results of third-order squeezing of the fundamental mode in spontaneous and stimulated interaction on a short-time scale in a six-wave mixing process, as reported by another author (Rani et. al., 2011):

$$\left[\left(\Delta\hat{Z}_{1\hat{A}}\right)^2-\frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2+9\hat{N}_{\hat{A}}+6\right\rangle\right]= -27|g|^2t^2\left(|\alpha|^8+2|\alpha|^6\right)\cos 6\theta \quad (44)$$

and

$$\left[\left(\Delta\hat{Z}_{1\hat{A}}\right)^2-\frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2+9\hat{N}_{\hat{A}}+6\right\rangle\right]= -\frac{27}{2}|g|^2t^2\left(|\beta|^4+4|\beta|^2+2\right)\left(|\alpha|^8+2|\alpha|^6\right)\cos 6\theta \quad (45)$$

where $|\alpha|^2=\left\langle \hat{A}^\dagger\hat{A}\right\rangle$, $|\beta|^2=\left\langle \hat{B}^\dagger\hat{B}\right\rangle$ and θ is the phase angle.

Using equations (44) and (45) in equation (43), we obtain respectively as

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2-\left(\frac{1}{4}\right)= -27|g|^4t^4m^4\left(|\alpha|^8+2|\alpha|^6\right)\cos 6\theta \quad (46)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2-\left(\frac{1}{4}\right)= -\frac{27}{2}|g|^4t^4m^4\left(|\beta|^4+4|\beta|^2+2\right)\left(|\alpha|^8+2|\alpha|^6\right)\cos 6\theta \quad (47)$$

The nonlinear factor $(|\beta|^4+4|\beta|^2+2)$ in equation (47) is due to the effect of stimulated interaction. This demonstrates that typical squeezing occurs in the signal mode. The findings also show that the squeezing in the stimulated process is larger than the corresponding squeezing in the spontaneous process. The maximum squeezing occurs when the value is $\theta \to 0$ and the minimum when the value is $\theta \to \pi/6$.

According to equations (44-47), it is inferred that squeezing in the signal mode is more than the fundamental mode if $g^2\, t^2\, m^4 > 1$ and if $g^2\, t^2\, m^4 < 1$, then corresponding squeezing is greater in the fundamental mode.

Now, compare the results of this paper's equations (32) and (33) with the quoted results of the following equations from previously published work (Choudhary & Giri, 2018);

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2-\left(\frac{1}{4}\right)=|g|^2t^2[\left(\Delta\hat{W}_2\right)^2-\left(\frac{1}{4}\right)\times \left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2+18\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}-2N_{\hat{B}}^3+6\hat{N}_{\hat{A}}^2-4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^3\right\rangle] \quad (48)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2-\left(\frac{1}{4}\right)=|g|^2t^2[\left(\Delta\hat{W}_1\right)^2-\left(\frac{1}{4}\right)\times \left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}^2+18\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}}-2N_{\hat{B}}^3+6\hat{N}_{\hat{A}}^2-4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}}^3\right\rangle] \quad (49)$$

Similarly, compare the findings of equations (42) and (43) in this study with the results of the following equations from previously published work (Choudhary & Giri, 2018);

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2-\left(\frac{1}{4}\right)= |g|^2m^6t^2\times\left[\left(\Delta\hat{Y}_{2\hat{A}}\right)^2-\left\langle\hat{N}_{\hat{A}}+\frac{1}{2}\right\rangle\right] \quad (50)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2-\left(\frac{1}{4}\right)= |g|^2m^6t^2\left[\left(\Delta\hat{Y}_{1\hat{A}}\right)^2-\left\langle\hat{N}_{\hat{A}}+\frac{1}{2}\right\rangle\right] \quad (51)$$

Due to the association with a large number of pump photons, the three-photon absorption six-wave interaction process produces more prominent squeezed laser light than the two-photon absorption six-wave interaction method (Choudhary & Giri, 2018).

IV. RESULTS AND DISCUSSION

We plot a graph (figure 2) between the left hand side of equation (32) or (33) say D_{SW} versus $|\alpha|^2$ with typical values $\left(\Delta\hat{W}_1\right)^2=\left(\Delta\hat{W}_2\right)^2=\left(\frac{1}{4}\right)$ to satisfy the equation (14).

The curves (figure 2) indicate that difference squeezing is present and is nonlinearly proportional to the quantity of pump photons. The degree of difference squeezing grows increasingly negative as the number of pump photons $|\alpha|^2$ increases until a threshold value of pump photon is achieved, after which it diminishes and eventually vanishes. The results are consistent with those of Truong et al. (Truong et. al., 2014). Furthermore, difference squeezing increases with the number of Stokes photons $|\beta|^2$. As a result, difference squeezing is found to be maximum at higher Stokes photon values, i.e., in stimulated emission rather than spontaneous emission.

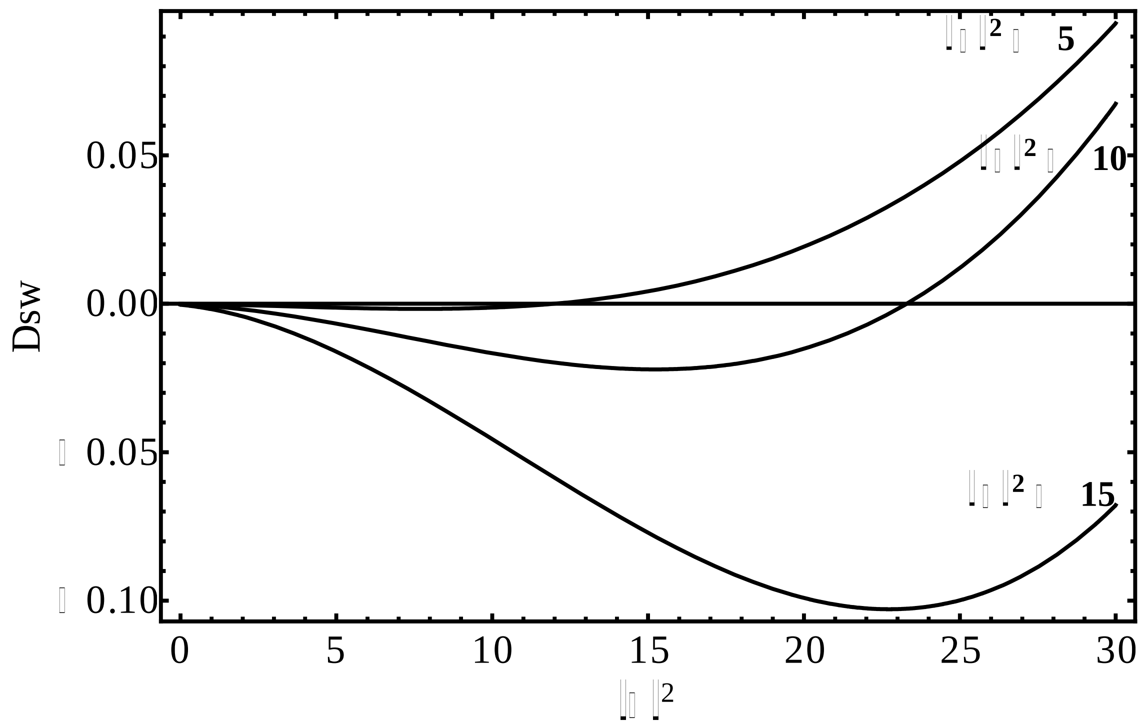

Figure 2: Degree of difference squeezing (D_{SW}) with $|\alpha|^2$ in degenerate three-photon absorption six-wave interaction process (when $|gt|^2 = 10^{-6}$ and $|\beta|^2 = 5, 10, 15$)

To investigate higher-order squeezing, denote the right hand side of equations (42) or (43) by $D_{SW'}$ and plot it $|gt|^2$ as shown in figure 3.

The constant decay of the curve (figure 3) suggests that the difference squeezing response is nonlinearly proportional to the quantity of pump photons. It indicates that difference squeezing increases as the number of pump photons increases. Additionally, difference squeezing grows as the number of pump photons $|\alpha|^2$ increases. As a result, it is noticed that difference squeezing is greatest when a high number of pump photons are assimilated. Thus, difference squeezing (higher-order squeezing) enables much more noise reduction than normal squeezing.

As shown in figure 4, compare equations (46) and (47) and construct a graph between squeezing S and $|\alpha|^2$ with various values of $|gt|^2$. As demonstrated in Figure 4, the squeezing in signal mode is stronger than the corresponding squeezing in pump mode (Rani et. al., 2011). It also confirms that the coupling of the field amplitude and interaction duration is strongly related to higher-order squeezing. As a result, more squeezed laser light may be detected in less time.

For arbitrary constant values of m^2 (stokes mode), we represent the right-hand side of equations (46) and (47) respectively by S_s and S'_s. and display the curve with $|\alpha|^2$ as shown in figures 5 (a) and 5 (b).

Figures 5 (a) and 5 (b) indicate that the squeezing rises nonlinearly with $|\alpha|^2$ and is directly proportional to the number of photons. We observe that when the Stokes photon value increases, the squeezing increases and the depth of classicality of the field amplitude diminishes in the stimulated process. It demonstrates that squeezing in signal mode is precisely proportional to the photon number of both the fundamental and Stokes modes. It also infer that, despite having the same number of photons, squeezing is greater in stimulated interaction than in spontaneous interaction.

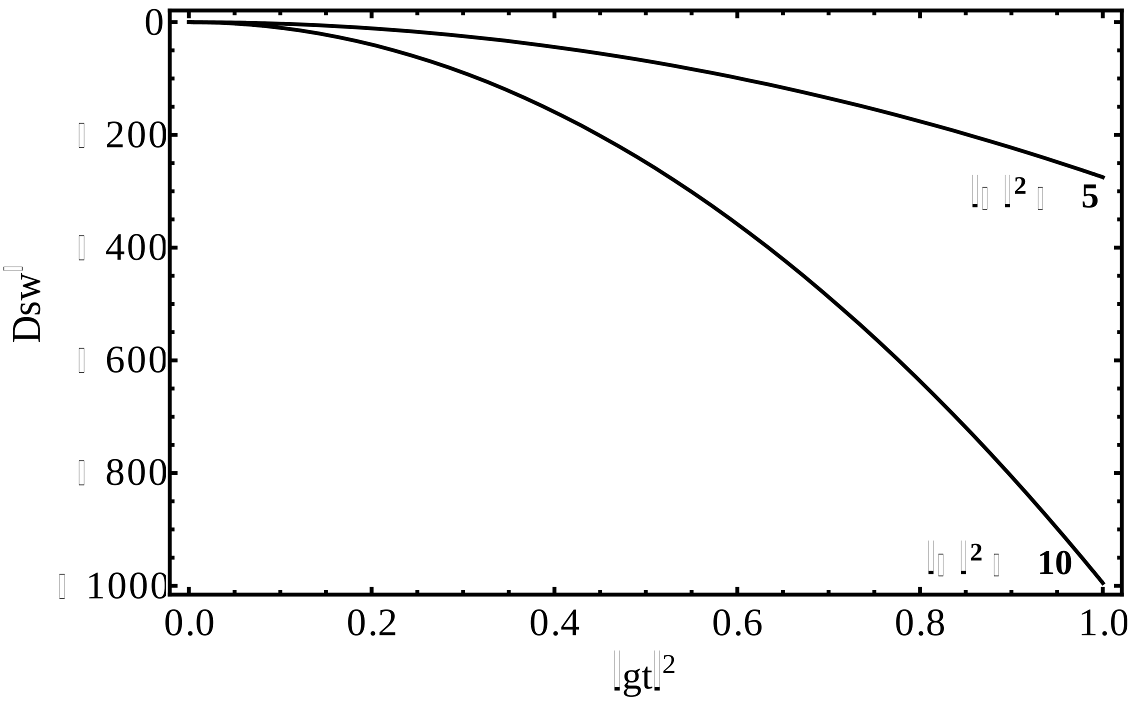

Figure 3: Degree of the squeezing $D_{SW'}$ with $|gt|^2$ (when $m^2 = 4$ = Constant) in degenerate three-photon absorption six-wave interaction process

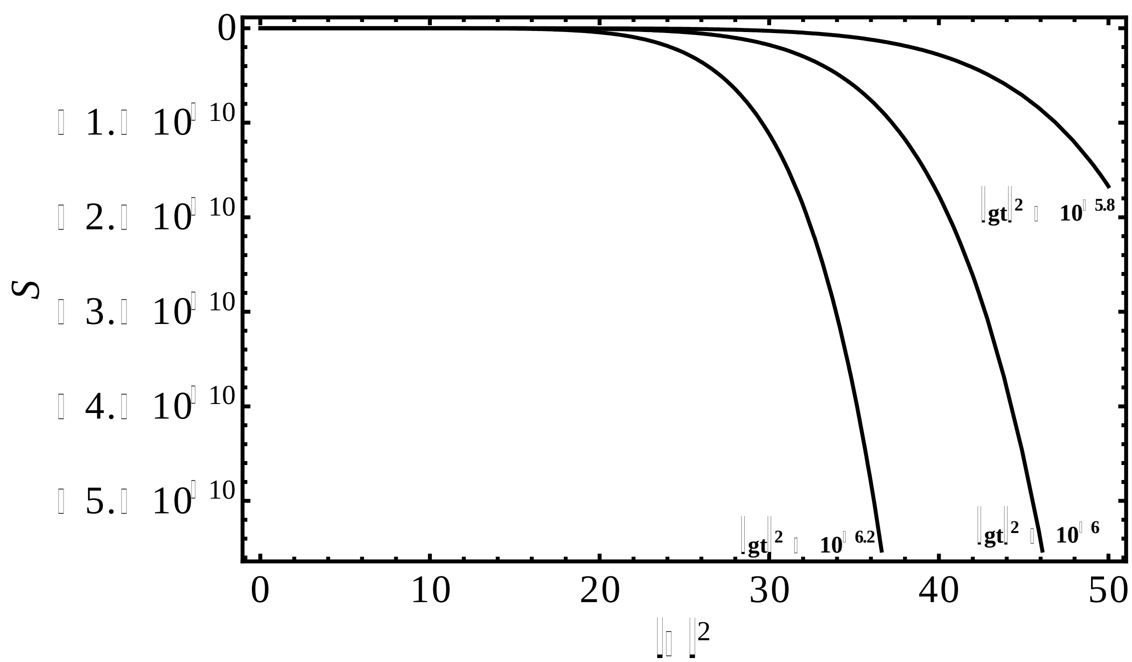

Figure 4: Degree of the squeezing (S) with $|\alpha|^2$ (when θ = 0, $|\beta|^2$=0 & m^2=1) in degenerate three-photon absorption six-wave interaction process

We draw a graph (figure 6) connecting the left-hand side of equation (48) or (49) say D_{SW}^2 with $|\alpha|^2$ having the same condition as in figure 2. Similarly, the right-hand side of equation (50) or (51) is represented by D_{SW}^3 and plotted with $|gt|^2$ as illustrated in figure 7.

When comparing figures 2 and 6, it can be seen that when the Stokes photon value rises, the depth of classicality of the field amplitude decreases. When comparing figures 3 and 7, it is found that the difference squeezing rises as the number of pump photons increases. As a result, the difference squeezing in the three-photon absorption six-wave interaction process is larger than the difference squeezing in the two-photon absorption six-wave interaction process (Choudhary & Giri, 2018). Therefore, the multi-photon absorption approach is superior for producing high-order squeezed light in an optical system.

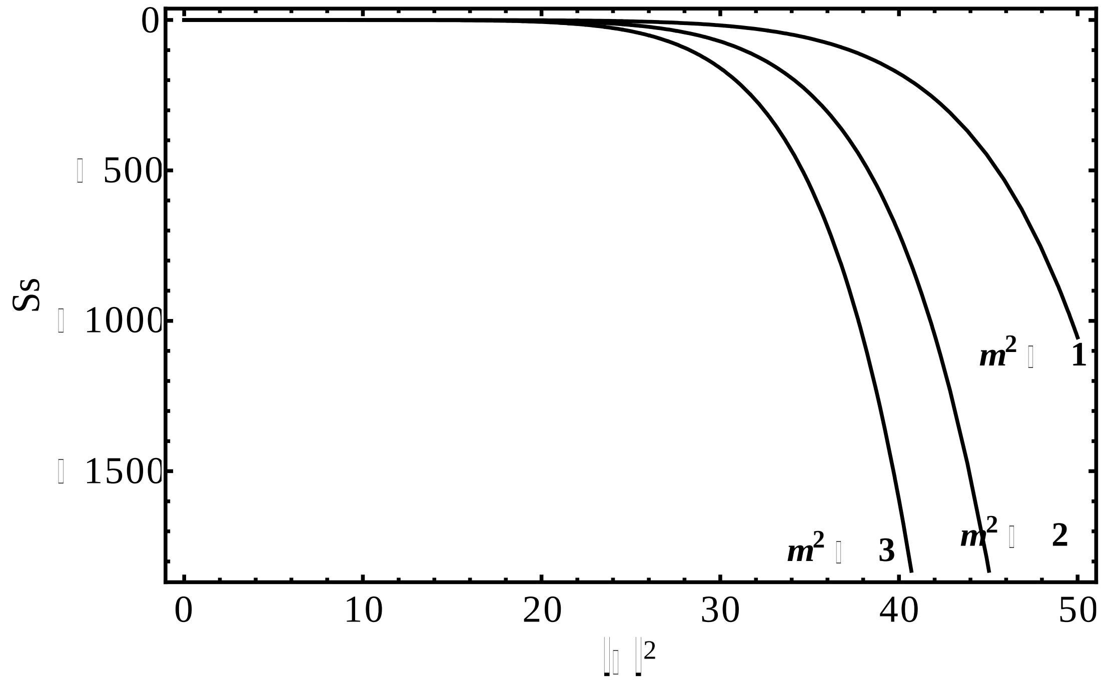

Figure 5 (a): Degree of the squeezing (Ss) in signal mode with $|\alpha|^2$ ($|\beta|^2$=0) in spontaneous degenerate three-photon absorption six-wave interaction process (when $|gt|^2 = 10^{-6}$ and θ = 0)

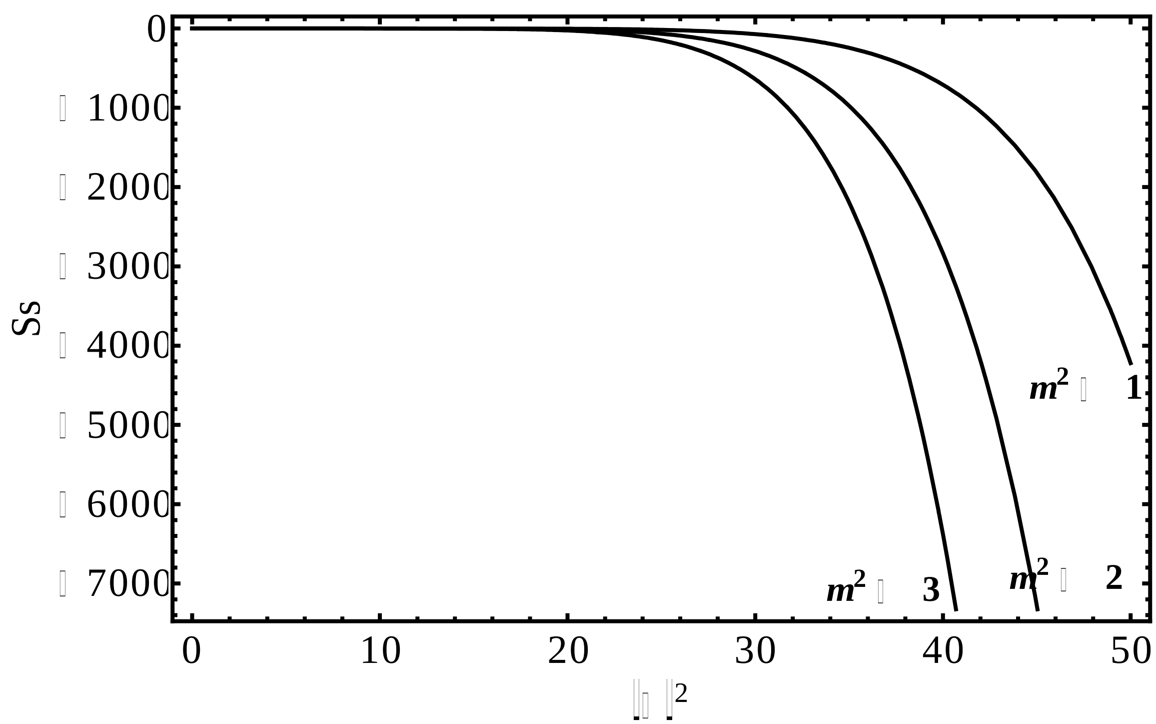

Figure 5 (b): Degree of the squeezing (Ss′) in signal mode with $|\alpha|^2$ ($|\beta|^2$=4) in stimulated degenerate three-photon absorption six-wave interaction process (when $|gt|^2 = 10^{-6}$ and θ = 0)

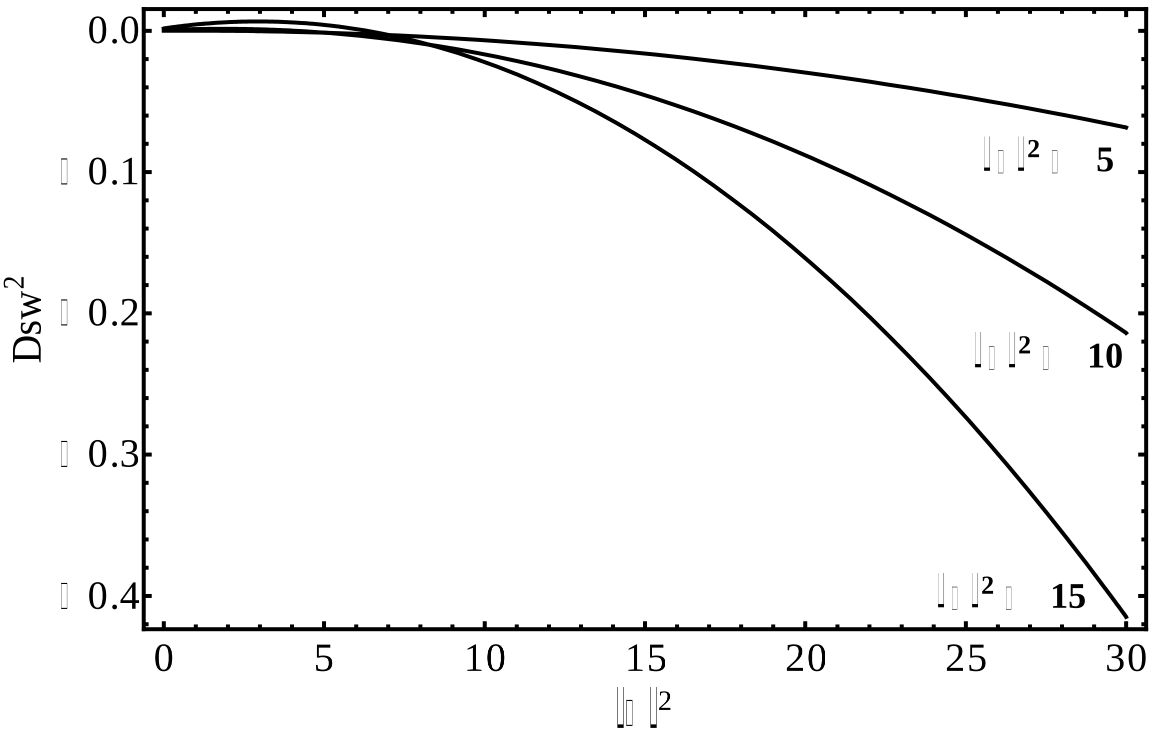

Figure 6: Degree of difference squeezing (D_{SW}^2) with $|\alpha|^2$ in degenerate two-photon absorption six-wave interaction process (when $|gt|^2 = 10^{-6}$ and $|\beta|^2 = 5, 10, 15$)

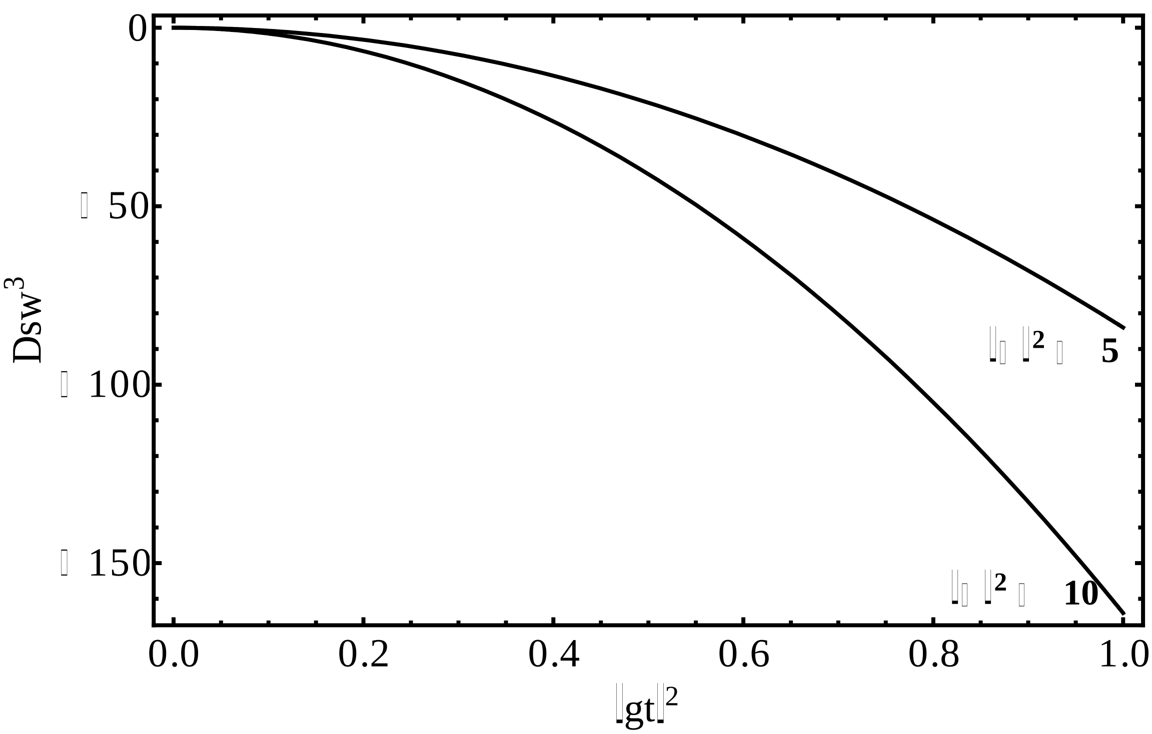

Figure 7: Degree of the squeezing (D_{SW}^3) with $|gt|^2$ (when $m^2 = 4$ = Constant) in degenerate two-photon absorption six-wave interaction process

V. SUMMARY AND CONCLUSIONS

In this paper, we have shown that difference squeezing for the uncorrelated mode between the pump and Stokes modes can be turned into normal squeezing in the signal mode and vice-versa. This result established the relationship between difference squeezing and normal squeezing and also suggests a method for detection of difference squeezing in the six-wave difference-frequency generation process. We confirmed that the squeezed states are associated with a large number of photons. It is found that when the number of photons is increasing, the degree of difference squeezing gets more negative until the critical value of the pump photon is reached, but subsequently, it decreases and finally disappears. As a result, difference squeezing occurs only in particular pump photon domain values. The results are consistent with those of Truong et al. (Truong et. al., 2014). The higher-order squeezing (difference squeezing) is found to be directly reliant on field amplitude coupling between modes and interaction time. Hillery's finding (Hillery, 1989) was confirmed.

We discovered that a squeezed signal mode (normal squeezing) is produced when a fundamental mode with amplitude-cubed squeezing (third-order squeezing) propagates through a nonlinear medium. Higher-order squeezing is converted to normal squeezing through nonlinear interaction (signal mode). In degenerate six-wave difference frequency generation, it proposes a method for detecting amplitude-cubed squeezing (third-order squeezing). In the present research, the squeezing obtained in the signal mode is determined to be larger than the corresponding squeezing in the fundamental mode (Rani et. al., 2011).

Furthermore, we discovered that when the value of Stokes photon grows, the squeezing increases and the depth of classicality of the field amplitude decreases in the stimulated process. Squeezing is observed to be greater in stimulated interaction than in spontaneous interaction, despite the fact that both have the same number of photons. The degree of squeezing in the signal mode is demonstrated to be exactly proportional to the photon number of both the fundamental and Stokes modes. We revealed that the difference squeezing is greater in the three-photon absorption six-wave interaction process than in the two-photon absorption six-wave interaction process (Choudhary & Giri, 2018). So, it can be concluded that the multi-photon absorption method is better for making high-order squeezed light in an optical system.

The findings of this paper are simple to replicate in most physical systems laboratories, paving the path for higher-order squeezing to be observed experimentally. Finally, in addition to these critical conceptual and foundational aspects, realizing larger and better multiphoton nonclassical states should open up new possibilities and perspectives for quantum optical realizations of quantum information and communication processes, which are currently unexplored (Dell'Anno et. al., 2006). These findings with higher-order nonlinearity also point to approaches for selecting appropriate processes for achieving better noise reduction in optical systems, which could be important in high-quality optical telecommunication and quantum information (Giri et. al., 2014).

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