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Principle of Physics and Total Fertility Rate

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Abstract: **Some social phenomenon were analysed by physicists using tools from core field of physics and the area of this type of study sometimes called socio-physics and econo-physics. Most of the work has appeared in physics journals. In the present paper an attempt has been made to develop this type of work in the field of demography. The focus is on the application of a differential equation model, to the study of the long term trend in level of fertility and infant mortality. The model is found to provide an excellent fit to the data, indicates that the trend is an exponential growth trend. Correlation and root mean squared error reveals that the estimated value has good agreement with the observed value. The model has been also used for prediction purpose.**

Index Terms: **Total fertility rate, Newton's law of cooling, Sample Registration System**

I. Introduction

Fertility measures the rate at which population adds to itself by births. Although the fertility is a biological process but it influences by social component also. Total Fertility Rate (TFR) which employs the synthetic cohort approach and indicates about how many children a female have had through their reproductive span. Thus, TFR is a measure of the average number of children ever born, a female will deliver during her childbearing age. The patterns of fertility are different for different countries and over the time also. Many of these factors that influence fertility are difficult to measure because they involve subjectivity and some of them may not apply across the cultures of the society. TFR is one of the demographic indicator that determine changes in tempo and quantum of the population thus if we require population estimates in the future, it is extremely essential to project this indicator. However, projection of the TFR has two inherent problems: first, we have to determine that value where the TFR will stabilize in future, which is called the stabilizer value or replacement level of fertility, and second, we have to determine the function which we are going to use to projecting the TFR. Estimating TFR is challenging for many developing countries because of limited data and varying data quality. India, the second most populous country in the world, is now experiencing the last stages of fertility transition. An increase in deliberate marital fertility control is observed when a population moves through the transition from natural to controlled fertility. Forecasting the demographic characteristics of a human population such as the fertility, mortality and migration is an important aspect of any socio-economic planning.

Fertility is considered one of the most important factors in the study of population dynamics. It refers to the actual reproductive outcomes and significantly affected by many demographic, socio-economic, cultural and biological factors. The demographers have given weight to understanding and define the fertility behaviour through large number of indirect techniques to estimate the TFR using exploratory variables. Brass (1968) suggested a P/F ratio method and Hobcraft et al. (1982) modified this. After that Cho et al. (1986) have suggested own child method which contains reverse survival technique (15 years) for estimating age specific fertility rate (ASFR) from cross-sectional survey. Furthermore stable population method has been used by Rele (1977) for estimating TFR's. With the use of sample registration system some modification has been done by Swamy et al. (1992). Coale and Demeny (1967) have developed a formula (TFR= P_3^2/P^2) to estimate the total fertility rate, where P^2 and P^3 represent mean births to females of age group 20-24 and 25-29. Gupta et al. (2014) modified considering situation of current time point. Yadava and Kumar (2002) have estimated TFR using percentage of currently married women having open birth interval greater than equal to 60 months. Jain (1997) has used Contraceptive prevalence rate (CPR) to estimate total fertility rate of any population. Mauldin and Ross (1991), Jain (1997) have used CPR to predict TFR and Singh et al. (2012) modified this model by taking the combination of CPR and sterility as a predictor variable to predict TFR of any population. It has been observed that there are various variables which affect the TFR. Most of the demographers developed models with single predictors or combination of two with coefficient of determination up to 0.9 or more to predict the TFR using cross sectional data. Very few studies have been available for longitudinal prediction of TFR.

Ševčíková et al. (2018) proposed probabilistic projection of TFR using autoregressive model in Bayesian paradigm and discuss various related models. González-Rosas et al. (2018) used stable bounded theory to predict TFR. Alkema et al. (2012) developed a method to

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estimate and observed uncertainty in TFR over the time based on multiple imperfect observations, local smoothing and weighted likelihood bootstrap. Saboia (1977) and McDonald (1979) used a time series model to predict total number of births. Miller (1986) employed a bivariate autoregressive model, which actually acted as a transfer function model, to forecast the total fertility and the mean age of childbearing. Further, Ortega and Poncela (2005) used a dynamic factor model with common and country-specific factors to forecast TFR. Lee (1993) modeled fertility over time using a single time-varying fertility index, that is, TFR. The author, however, observed that the long term fertility forecasts yielded in a large width of its prediction interval. Further, Lee and Tuljapurkar (1994) used the same model with a different value for the average level of TFR and they imposed no restriction on the limits of its bounds. Singh et al. (2017) tryied to relate TFR and infant mortality rate (IMR) using time series methodologies and found TFR is regulated by IMR. Tripathi et al. (2018) used Bayesian ARIMA methodology to predict TFR in India.

II. Data

The civil registration system in India has a lot to be improved and would require a huge effort and time. A need for having an alternate source of such information was required. The Government of India, in the late 1960s, initiated the Sample Registration System (SRS). The SRS is a demographic survey for providing reliable annual estimates of total fertility rate, infant mortality rate, birth rate, death rate and other fertility and mortality indicators at the national and sub-national levels. SRS ensure the completeness of vital events reporting using a representative sample of the population and based on dual-record system.

We have used year wise SRS data on TFR of Uttar Pradesh and India from 1971 to 2020. Also we know that the replacement level fertility is 2.1. Table 1 shows the data on TFR for India and Uttar Pradesh. In Uttar Pradesh, the most populous state of India has TFR during the early 70's around 6.6 which declined to 2.7 in year 2020. This shows the TFR in Uttar Pradesh has declined by approximately 59 percent in this period. Similarly India's TFR decline by 58 percent approximately (from 5.2 in 1971 to 2.2 in 2020).

Table 1: Observed TFR for India and Uttar Pradesh from SRS Reports

Year	TFR		Year	TFR	
	India	Uttar Pradesh		India	Uttar Pradesh
1971	5.2	6.6	1996	3.4	4.9
1972	5.2	6.6	1997	3.3	4.8
1973	4.9	6.4	1998	3.2	4.6
1974	4.9	6.4	1999	3.2	4.7
1975	4.9	6.6	2000	3.2	4.7
1976	4.7	5.9	2001	3.1	4.5
1977	4.5	6.1	2002	3.0	4.4
1978	4.5	6.0	2003	3.0	4.4
1979	4.4	5.8	2004	2.9	4.4
1980	4.4	5.9	2005	2.9	4.2
1981	4.5	5.8	2006	2.8	4.2
1982	4.5	5.7	2007	2.7	3.9
1983	4.5	5.8	2008	2.6	3.8
1984	4.5	5.9	2009	2.6	3.7
1985	4.3	5.6	2010	2.5	3.5
1986	4.2	5.4	2011	2.4	3.4
1987	4.1	5.5	2012	2.4	3.3
1988	4.0	5.4	2013	2.3	3.1
1989	3.9	5.2	2014	2.3	3.2
1990	3.8	5.2	2015	2.3	3.1
1991	3.6	5.1	2016	2.3	3.1
1992	3.6	5.2	2017	2.2	3.0
1993	3.5	5.2	2018	2.2	2.9
1994	3.5	5.1	2019	2.2	2.8
1995	3.5	5.0	2020	2.2	2.7

III. Methodology

Researchers of physics have using some methodologies from physics to study some phenomena considered to fall within the domain of the social sciences, and these attempts sometimes referred to as socio-physics and/or econo-physics. The work of physicists on social networks, collective decision making, financial issues, and income distribution (Toivonen, et al., 2006; de Silva, et al., 2006; and Clement and Gallegati, 2005; Galam, 1997) are some key examples. Even though some social scientists have been also open to this effort (Ormerod and Colbaugh, 2006 & Keen and Standish, 2006), and it is right to say that the influence of these type of works have probably been felt more within the physics than outside of it. The work on social scientific questions by physicists should be encouraged because the social studies are complex enough and need simple solution. We also think an approach of using tools from physics to study social issues should become more visible and attract social scientists. The present paper focuses on the application of a differential equation model that has been used in financial theory and theories of economic growth but which comes up very frequently in physics and physical chemistry. In Physics there are some example such as models of radioactive decay, Newton's Law of Cooling and changes in the concentration of reactants over time for a first order reaction. What, all these examples have in common is that the rate of change in some quantity over time is proportional to the amount of that quantity.

Thus, in Newton's Law of cooling the temperature of something is decreasing over time. This example of cooling is seems to be relevant to the topic of this paper i.e. to explaining the pattern of TFR, since it is concerned with decrease in the TFR level over time. In fact, knowledge through education, use of contraceptive and desire of less number of children may be consider as a factors that decrease in the TFR of the society. In any case, it will be shown that the TFR level decreases over time in a way analogous to decreases in temperature in accordance with Newton's Law of Cooling.

Newton's Law of Cooling

A standard physics model attributed to Sir Isaac Newton, as a result of some experimental work he had done, is known as Newton's Law of Cooling. In equation form, the law states that:

$$\frac{dT}{dt} = k(T - T_s) \tag{1}$$

Where 'T' denotes the temperature of a given object, 't' denotes time, 'T_s' denotes the temperature of the surrounding environment, and 'k' is a constant of proportionality.

Equation (1) is an example of an ordinary differential equation that can be solved by the method of separating variables. If both sides of equation (1) are divided by ($T - T_s$) and multiplied by dt, we get:

$$\frac{dT}{(T - T_s)} = kdt \tag{2}$$

If we integrate both sides of equation (2), We get:

$$Ln(T - T_s) = kt + C \tag{3}$$

Where 'C' is an arbitrary constant. Taking the antilogarithms of both sides of equation (3) leaves us with:

$$(T - T_s) = e^{kt+C} = e^{kt}e^{C} \tag{4}$$

Taking this into account and adding T_s to both sides of equation (4) gives us:

$$T = Ae^{kt} + T_s \tag{5}$$

Where $A = e^C$. This equation (5) is the general solution of equation (1).

If 'k' is less than zero, equation (5) tells us how the temperature of an object, surrounded by an environment at T_s, will decrease over time until it reaches the same temperature as its environment. The mechanism behind Newton's law of cooling has to do with the second law of thermodynamics, which, in one formulation, stipulates that heat always flows from a higher temperature to a lower temperature.

Now we assume that the changes in the fertility level over time are proportional to the level of fertility at a given time. Symbolically this is:

$$\frac{dF}{dt} = k(F - F_r) \tag{6}$$

Here 'F' denotes the overall fertility level i.e TFR, F_r is the replacement level fertility, 't' denotes time, and k is a constant of proportionality. This equation, like equation 1, can be solved by separating variables. If both sides of equation (6) are multiplied by dt and both sides are divided by ($F - F_r$), we end up with:

$$\frac{dF}{(F - F_r)} = kdt \tag{7}$$

If we now integrate both sides of equation (7), we get:

$$Ln(F - F_r) = kt + C \tag{8}$$

where 'C' is an arbitrary constant. Taking the antilogarithms of both sides of equation (8) leaves us with:

$$(F - F_r) = e^{kt}e^{C} \tag{9}$$

If we replace e^C with A we get: $(F - F_r) = Ae^{kt}$ (10)

$$\Rightarrow Ln(F - F_r) = LnA + kt \tag{11}$$

We can estimate A and k by least square estimation procedure.

IV. Piecewise Linear Regression

Piecewise regression is a special type of linear regression that arises when a single line is not sufficient to model a data set. Piecewise regression breaks the phenomenon into many segments and fits a separate line for each one. The basic idea behind piecewise linear regression is that if the data follow different linear trends over time of the data then we should model the regression function in pieces. The pieces may be connected or not connected. Here, we will fit a model in which two pieces are connected. The data set contains information on the TFR (y) and the corresponding year (x). When analyzing a relationship between a response y, and an explanatory variable x, it may be obvious that for different ranges of x, different linear relationship observes. In these cases, neither a single linear model provides an adequate explanation nor an appropriate nonlinear model find out. Breakpoints are the values of x where the slope of the linear function changes. The value of the breakpoint may or may not be known before the analysis, but typically it is unknown and must be estimated or guess through graph. The regression function at the breakpoint may be discontinuous, but a model can be written in such a way that the function is continuous at all points including the breakpoints.

V. Results

Table 2 reports the results based on the equation (11) overall as well as piecewise regression model for Uttar Pradesh from 1971 to 2020. For overall duration i.e., 1971 to 2020, the model shows a negative significant regression coefficient (-0.035) with R^2 (0.886). The value of intercept is 1.763 which is also significant. This model explains TFR about 90 percent. The Negative regression coefficient shows as the year increases the TFR decreases with rate 0.035 per year in Uttar Pradesh. The piecewise regression shows an improved result. For the two segments i.e., 1971 to 2004 and 2005 to 2020, the value of R^2 is 0.969 and 0.976 respectively. The result of piecewise regression explains TFR almost 9 percent more than overall regression. In the first segment i.e., from 1971 to 2004, regression coefficient is -0.020 with intercept value 1.539 and both values are statistically significant. According to the regression for 1971 to 2004, a 10-year increase is associated with a decline of 0.20 births per woman. On the other hand, for the second segment of time i.e., 2005 to 2020, regression coefficient is -0.078 with intercept value of 3.526. These coefficients are also statistically significant. An increase of 10 years, the regression slop shows a decrement of 0.78 birth per women in the TFR. The Standard error of R^2 for overall model is 0.186 whereas for models of 1971 to 2004 and 2005 to 2020, it is 0.036 and 0.061 respectively. The lower value standard error shows a better consistent result of regression.

Table 2: Model Summary for Uttar Pradesh

Range	Model	Coefficient	*t*-value	*p*-value	R^2	Standard error
1971-2020	Year	-0.035	-19.359	0.000	0.886	0.186
	Intercept	1.763	32.946	0.000		
1971-2004	Year	-0.020	-31.818	0.000	0.969	0.036
	Intercept	1.539	120.794	0.000		
2005-2020	Year	-0.078	-23.938	0.000	0.976	0.061
	Intercept	3.526	25.030	0.000		

In Figures the graph of natural logarithm of TFR according to year and fitted regression line over the TFR data from year 1971 to 2020 of Uttar Pradesh is shown. For overall duration i.e., form year 1971 to 2020, Figure 1 shows considerable deviation from the regression lines in a number of data points. The graph clearly shows two different linear trends so piecewise regression should be a better choice to explain data. Figure 2 plots regression line for the first segment of time duration i.e., form year 1971 to 2004 and Figure 3 shows regression line for the second segment of time duration i.e., form year 2005 to 2020 in Uttar Pradesh. Each figure shows a good fitting of regression line in comparison to overall (Figure 1).

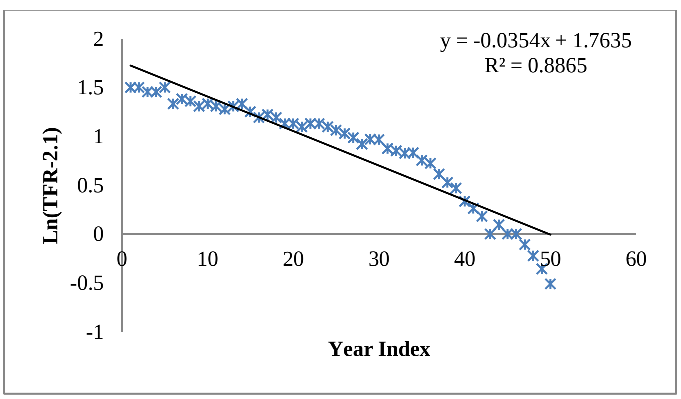

Figure 1: Trend of TFR over the time (from 1971 to 2020) in Uttar Pradesh

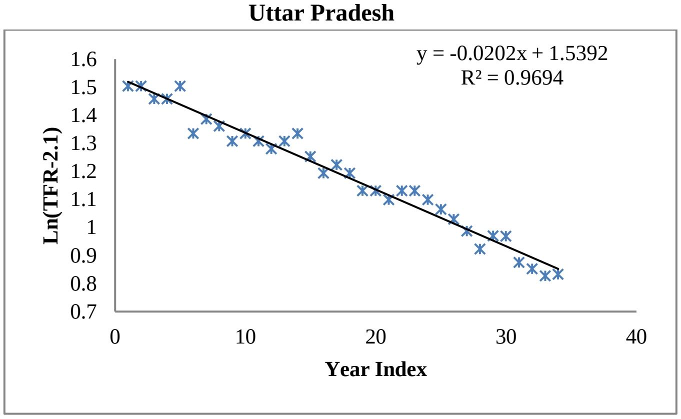

Figure 2: Trend of TFR over the time (from 1971 to 2004) in Uttar Pradesh

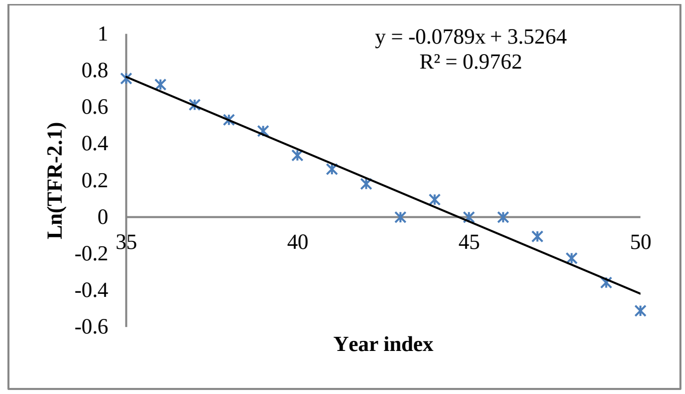

Figure 3: Trend of TFR over the time (from 2005 to 2020) in Uttar Pradesh

Table 3 reports observed and estimated TFR for Uttar Pradesh from 1971 to 2020 along with the confidence intervals. There are almost same results of estimated TFR against observed TFR shows the suitability of the model used. Figure 4 shows a graph of observed and expected TFR in Uttar Pradesh. The correlation is also very high i.e., 0.996 and hence shows a good fitting.

Table 3: Observed and Estimated TFR for Uttar Pradesh

Time	TFR	Ln(TFR-2.1)	Estimated TFR	95% Confidence Interval	
				Lower	Upper
1971	6.6	1.50	6.67	6.56	6.78
1972	6.6	1.50	6.58	6.47	6.68
1973	6.4	1.46	6.49	6.39	6.59
1974	6.4	1.46	6.40	6.31	6.49
1975	6.6	1.50	6.31	6.23	6.40
1976	5.9	1.34	6.23	6.15	6.31
1977	6.1	1.39	6.15	6.07	6.22
1978	6.0	1.36	6.06	6.00	6.14
1979	5.8	1.31	5.99	5.92	6.05
1980	5.9	1.34	5.91	5.85	5.97
1981	5.8	1.31	5.83	5.78	5.89
1982	5.7	1.28	5.76	5.70	5.81
1983	5.8	1.31	5.68	5.63	5.73
1984	5.9	1.34	5.61	5.57	5.66
1985	5.6	1.25	5.54	5.50	5.59
1986	5.4	1.19	5.47	5.43	5.52
1987	5.5	1.22	5.41	5.36	5.45
1988	5.4	1.19	5.34	5.30	5.38
1989	5.2	1.13	5.27	5.23	5.32
1990	5.2	1.13	5.21	5.17	5.25
1991	5.1	1.10	5.15	5.11	5.19
1992	5.2	1.13	5.09	5.05	5.13
1993	5.2	1.13	5.03	4.99	5.07
1994	5.1	1.10	4.97	4.93	5.01
1995	5.0	1.06	4.91	4.87	4.96
1996	4.9	1.03	4.86	4.81	4.90
1997	4.8	0.99	4.80	4.75	4.85
1998	4.6	0.92	4.75	4.70	4.80
1999	4.7	0.97	4.69	4.64	4.75
2000	4.7	0.97	4.64	4.59	4.69
2001	4.5	0.88	4.59	4.54	4.65

2002	4.4	0.85	4.54	4.49	4.60
2003	4.4	0.83	4.49	4.44	4.55
2004	4.4	0.83	4.44	4.39	4.50
2005	4.2	0.76	4.25	4.12	4.39
2006	4.2	0.72	4.09	3.98	4.20
2007	3.9	0.61	3.94	3.84	4.03
2008	3.8	0.53	3.80	3.72	3.88
2009	3.7	0.47	3.67	3.60	3.73
2010	3.5	0.34	3.55	3.50	3.60
2011	3.4	0.26	3.44	3.39	3.49
2012	3.3	0.18	3.34	3.30	3.38
2013	3.1	0.00	3.24	3.21	3.28
2014	3.2	0.10	3.16	3.12	3.19
2015	3.1	0.00	3.08	3.04	3.11
2016	3.1	0.00	3.00	2.97	3.04
2017	3.0	-0.11	2.93	2.90	2.97
2018	2.9	-0.22	2.87	2.83	2.91
2019	2.8	-0.36	2.81	2.77	2.85
2020	2.7	-0.51	2.76	2.72	2.80

Table 4: Projected TFR for Uttar Pradesh

Year	Projected TFR	95% Confidence Interval	
		Lower	Upper
2021	2.71	2.67	2.75
2022	2.66	2.62	2.71
2023	2.62	2.58	2.66
2024	2.58	2.54	2.62
2025	2.54	2.50	2.59
2026	2.51	2.47	2.55
2027	2.48	2.44	2.52
2028	2.45	2.41	2.49
2029	2.42	2.39	2.47
2030	2.40	2.36	2.44

In Table 4 the projected values of TFR from 2021 to 2030 is given for Uttar Pradesh. It shows rapid decline in TFR i.e., from 2.71 in 2021 to 2.40 in 2030.

Table 5: Model Summary for India

Range	Model	Coefficient	*t*-value	*p*-value	R^2	Standard error
1971-2020	Year	-0.061	-19.370	0.000	0.885	0.353
	Intercept	1.569	16.439	0.000		
1971-2000	Year	-0.036	-23.271	0.000	0.950	0.074
	Intercept	1.221	44.109	0.000		
2001-2020	Year	-0.136	-23.013	0.000	0.967	0.153
	Intercept	4.439	18.278	0.000		

Table 5 reports the overall as well as piecewise regression model for India from 1971 to 2020. Overall regression coefficient is -0.061 with intercept value 1.569. Therefore, it can be concluded that the regression line measures the negative change in TFR for each one-year increase. In other words, a 10 time-point increase is associated with 0.61 decreases in TFR of India. In regression model for first segment of overall duration i.e., from the year 1971 to 2000 for India shows a better value of R^2 with regression coefficient -0.036 and intercept value 1.221. For the second segment of overall duration i.e., form year 2001 to 2020 for India, the regression coefficient is -0.136 with intercept value 4.439. Thus, for second segment of time the model shows a 10 time-point increase is associated with a decrease of 0.136 in TFR. The value of R^2 from piecewise regression implies a gain of 7 percent in explanation and fitting of data. It has been observed that as far as TFR reaches closer to the replacement level fertility (2.1) the rate of change become slower.

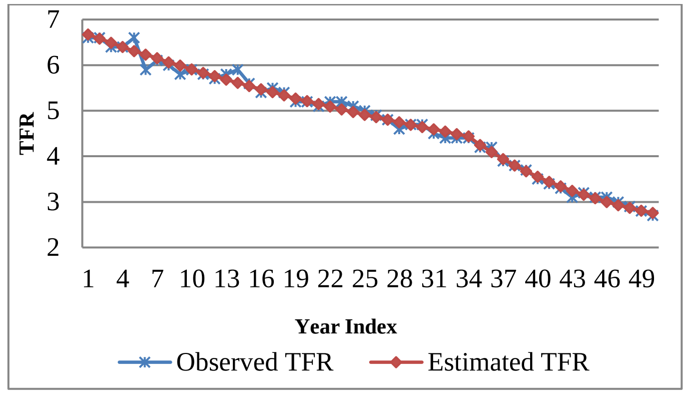

Figure 4: Observed and Expected TFR in Uttar Pradesh (Correlation is 0.996)

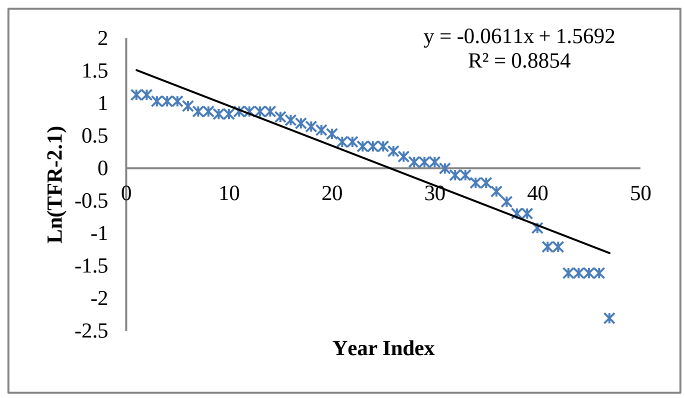

Figure 5: Trend of TFR over the time (from 1971 to 2020) in India

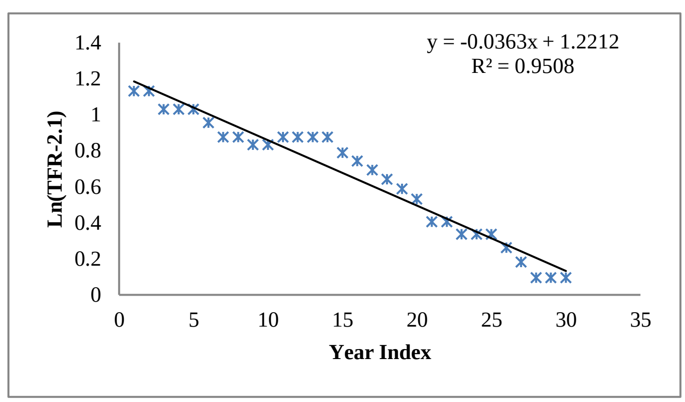

Figure 6: Trend of TFR over the time (from 1971 to 2000) in India

Figure 5 shows fitted regression line for India over the time from 1971 to 2020 but a number of time points deviated from it. Figure 6 shows fitted regression line for first segment of overall time duration from year 1971 to 2000 for India. Figure 7 shows fitted regression line for second segment of overall time duration from year 2000 to 2020 for India. Figure 6 and 7 shows a better fitting of regression line than Figure 5.

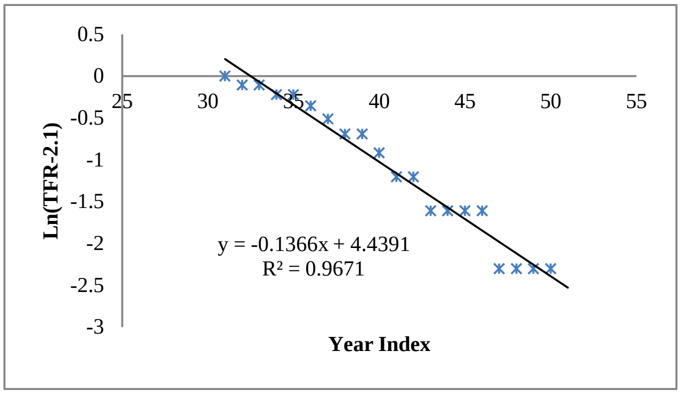

Figure 7: Trend of TFR over the time (from 2001 to 2020) in India

Table 6 reports Observed and Estimated TFR for India from 1971 to 2020. The Estimated TFR confirms the observed value up to an extent. The predicted results are very exciting. Figure 8 represents the graph of Observed and Estimated TFR for India and correlation between them is 0.991 which is very promising and hence the suitability of piecewise regression is obvious.

Table 7 reports the projection of TFR from 2021 to 2030. It shows a decreasing trend i.e., from 2.18 in year 2021 to 2.12 in year 2030. The confidence interval for the estimated TFR is also very narrow which shows a better degree of confidence for projected values.

Table 6: Observed and Estimated TFR for India

Time	TFR	Ln(TFR-2.1)	Estimated TFR	95% Confidence Interval	
				Lower	Upper
1971	5.2	1.13	5.37	4.88	5.94
1972	5.2	1.13	5.25	4.79	5.80
1973	4.9	1.03	5.14	4.69	5.67
1974	4.9	1.03	5.03	4.60	5.54
1975	4.9	1.03	4.93	4.52	5.41
1976	4.7	0.96	4.83	4.43	5.29
1977	4.5	0.88	4.73	4.35	5.18
1978	4.5	0.88	4.64	4.27	5.06
1979	4.4	0.83	4.55	4.19	4.96
1980	4.4	0.83	4.46	4.12	4.85
1981	4.5	0.88	4.38	4.05	4.76
1982	4.5	0.88	4.29	3.98	4.66
1983	4.5	0.88	4.22	3.91	4.57
1984	4.5	0.88	4.14	3.85	4.48
1985	4.3	0.79	4.07	3.79	4.40
1986	4.2	0.74	4.00	3.73	4.31
1987	4.1	0.69	3.93	3.67	4.23
1988	4.0	0.64	3.86	3.61	4.16
1989	3.9	0.59	3.80	3.56	4.09
1990	3.8	0.53	3.74	3.51	4.02
1991	3.6	0.41	3.68	3.46	3.95
1992	3.6	0.41	3.63	3.41	3.88
1993	3.5	0.34	3.57	3.36	3.82
1994	3.5	0.34	3.52	3.31	3.76
1995	3.5	0.34	3.47	3.27	3.70
1996	3.4	0.26	3.42	3.23	3.65
1997	3.3	0.18	3.37	3.19	3.59
1998	3.2	0.10	3.33	3.15	3.54
1999	3.2	0.10	3.28	3.11	3.49
2000	3.2	0.10	3.24	3.07	3.44
2001	3.1	0.00	3.23	3.11	3.41
2002	3.0	-0.11	3.17	3.04	3.32
2003	3.0	-0.11	3.03	2.93	3.15
2004	2.9	-0.22	2.91	2.83	3.01
2005	2.9	-0.22	2.81	2.74	2.88
2006	2.8	-0.36	2.72	2.67	2.78
2007	2.7	-0.51	2.64	2.60	2.69
2008	2.6	-0.69	2.57	2.54	2.61
2009	2.6	-0.69	2.51	2.48	2.54
2010	2.5	-0.92	2.46	2.43	2.49
2011	2.4	-1.20	2.41	2.39	2.44
2012	2.4	-1.20	2.37	2.35	2.39
2013	2.3	-1.61	2.34	2.32	2.36
2014	2.3	-1.61	2.31	2.29	2.33
2015	2.3	-1.61	2.28	2.27	2.30
2016	2.3	-1.61	2.26	2.24	2.27
2017	2.2	-2.30	2.24	2.22	2.25
2018	2.2	-2.30	2.22	2.21	2.24
2019	2.2	-2.30	2.20	2.19	2.22
2020	2.2	-2.30	2.19	2.18	2.21

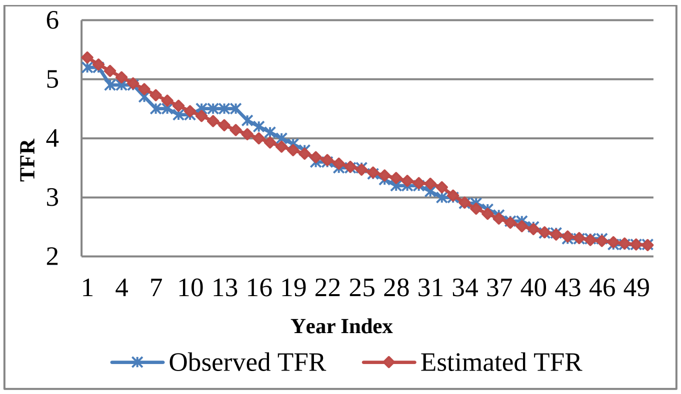

Figure 8: Observed and Expected TFR in India (Correlation is 0.991)

Table 7: Projected TFR for India

Year	Projected TFR	95% Confidence Interval	
		Lower	Upper
2021	2.18	2.17	2.19
2022	2.17	2.16	2.18
2023	2.16	2.15	2.17
2024	2.15	2.14	2.16
2025	2.15	2.14	2.16
2026	2.14	2.13	2.15
2027	2.14	2.13	2.14
2028	2.13	2.12	2.14
2029	2.13	2.12	2.14
2030	2.12	2.11	2.13

VI. CONCLUSION

We have modeled the long term dynamics of fertility level changes on the assumption that increases in education, autonomy of female in the society and desire to have less number of children respond to decrease in the fertility level. These assumptions led to an ordinary differential equation that is popular in physics for modeling the cooling of objects in a surrounding medium, and this model is found to have an excellent fit to a time series of fertility data for the Uttar Pradesh. We have proposed a new approach based on Newton's law of cooling for estimating the TFR over time and applied it on the data of Uttar Pradesh and India. The simplicity of the proposed method is that, it provides very close predicted values of TFR of Uttar Pradesh and India. The proposed technique is simple in terms of understanding and computing but the result is very surprising and very similar to the result given in Tripathi et al. (2018), however, they used very complicated Bayesian techniques. It is expected that such an analysis will help the policy and program makers to get at least an approximate idea of future fertility trend.

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