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Investigation of Cosmological Models using EoS parameter in Modified theory of Gravity

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Abstract: **The paper presents the study of Friedmann – Lemaitre – Robertson – Walker (FLRW) universe with the support of *F(T)* gravity, generalisation of Teleparallel Equivalent of General Relativity (TEGR). We have examined some models corresponding to the study of equation of state parameter (EoS) representing the different phases of the universe. Also, by assuming suitable functions for *F(T)*, we studied corresponding expression for energy density and pressure.**

Keywords:***F(T)*theory of gravity, FLRW spacetime, Equation of State parameter, Cosmological models, Energy density and pressure.**

I. Introduction

For understanding the nature of dark energy (DE) an alternative structure has been provided by an amended gravitation theory. This is done through the modifications of gravitational Lagrangian by introducing different functions of Ricci scalar, modifications of Hilbert-Einstein action have been systematically analysed (Nojiri et al. (2010) and Durrer et al. (2010)).

In the last two decades some prime modifications of Einstein's Theory of relativity has been proposed in which mostly discussed theories are *F(R)* theory of gravity and *F (R,T)*theory of gravity (where *R* is Ricci scalar and *T*is trace of energy momentum tenser)(Nojiri 2007,2009,2009a,2009b,Cognola et al.(2009),Elizalde et al.(2009)). The cosmological and astrophysical implications of the *F(R,T)* gravity theory as well as their properties have been investigated in(Sahoo(2018) and Wu et al. (2018)).On the other hand, *F(T)*theory of gravity is another theory which has gained popularity in last few years, gives good explanation of late time acceleration(Ferraro and Fiorini (2007),Bengochea et al. (2009),Linder(2010)). The *F(T)* theories of gravity are the generalisation of Teleparallel Equivalent of General Relativity (TEGR) (Cai et al. (2016)), which was first presented by Albert Einstein. *F(T)* gravity theories admit the early time inflation of the universe without referring to dark energy (DE) and various cosmological properties of modified *F(T)* gravity models have been discussed in (Linder (2010) and Bengochea (2009)). In GR the field equations consisting fourth order equations while the *F(T)*theories are having always equations of motion of second order. Observational solar system constrains (Iorio et al. (2012), (2015), Farrugia(2016)), cosmological perturbations(Dent et al. (2011), Zheng et al. (2011),Izumi et al. (2013),Li et al. (2011), Basilakos et al. (2016)),cosmological constrains(Bengochea et al. (2011),Wei et al. (2011),Capozziello et al. (2015),Oikonomou et al. (2016),Nunes et al. (2016)),the existence of relativistic stars (Böhmer et al. (2011), energy condition bounds (Liu and Reboucas (2012)), cosmographic constrains (Capozziello et al. (2011)) are some articles in which several qualities of *F(T)*gravity has been investigated. In (Otalora and Reoucas, (2017)) violation of causality in *F(T)*gravity has been studied. The acceleration of the universe can be appreciated by considering a *F(T)* gravity model in (Myrzakulov (2010)). In this work we are focused to determine definite solutions of the field equations in *F(T)*gravity for a flat homogeneous and isotropic FLRW spacetime. Also, we construct *F(T)*for a particular value of the EoS parameter. Equation of state (EoS) shows up the microscopic structure of the fluids at microscopic level via relations, that link together the thermos dynamical parameters of the fluid. A very simple equation of state (EoS) i.e $p = \omega \rho$, (here ω is

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constant) characterised a perfect fluid which has a particular importance in cosmology. Among this class of fluids, there are three special cases such as$\omega = 0, \omega = \frac{1}{3}, \omega = -1$. The first case $\omega = 0$ describes a gas of non-relativistic particles (whose kinetic energy is negligible compared to their rest energy).

The case $\omega = \frac{1}{3}$ is suitable to describe a gas of ultra-relativistic particles (whose rest energy is negligible with respect to their kinetic energy) finally the case $\omega = -1$ can be used to describe the so-called vacuum energy (FulvioSbis$\dot{a}(2014)$).

The composition of paper is as follows. In the Section II we have presented some basics of *F(T)*gravity model. In the Section III, we have done with *F(T)*gravity in a flat FLRW Space time with gravitational field equations. The construction of *F(T)*for distinct values of the EoS parameterω is performed in Section IV. In Section V, we find expressions for ρ and p by assuming *F(T)* is given. Finally, Section VI gives the conclusions and discussions of the results established.

II. The *F(T)*gravity model

The*F(T)*theory of gravity is described in Weitzenb$\ddot{o}$ck space-time with line element as specified by

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \quad (1)$$

here$g_{\mu\nu}$arethe metric components which are symmetric having 10 degrees of freedom.

The theory may be described in space time or in tangent space through the matrix, that permits to revised the line element (1) such that

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = \eta_{ij} \theta^i \theta^j, \quad (2)$$

$$dx^{\mu} = e_i^{\ \mu}\, \theta^{\ i}\, , \theta^i = e_{\mu}^{i}\, dx^{\ \mu}, \quad (3)$$

Where $e_i e_j = \eta_{ij}, \eta_{ij} = diag(1, -1, -1, -1)$ gives the Minkowski metric. The metric determinant under the radical sign is expressed as$\sqrt{-g} = \det[e_{\mu}^{i}] = e$ and the metric e_{μ}^{i} are called tetrad representing the dynamic fields of theory.

Let us examine the below action towards *F(T)* gravity as

$$S = \frac{1}{2k^2} \int dx^4 \left[\sqrt{-g} F(T) + L_m\right], \quad (4)$$

Here T denotes the torsion scalar and *F(T)*represents general differentiable function in relation to the torsion T and $L_m$corresponds to the matter Lagrangian and $k^2 = 8\pi G$.The elements of the tensor torsion are expressed by

$$T_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\omega\lambda} - \Gamma_{\mu\nu}^{\omega\lambda} = e_i^{\lambda}\left(\partial_{\mu} e_{\nu}^{i} - \partial_{\nu} e_{\mu}^{i}\right). \quad (5)$$

We now define two tensors in this gravity, the contortion tensor and antisymmetric tensor as

$$K_{\rho}^{\mu\nu} = -\frac{1}{2}(T^{\ \mu\nu}_{\ \rho} - T^{\ \nu\mu}_{\ \rho} - T_{\ \rho}^{\ \mu\nu}), \quad (6)$$

$$S_{\rho}^{\mu\nu} = \frac{1}{2}\left(K_{\rho}^{\mu\nu} + \delta_{\rho}^{\mu} T_{\theta}^{\theta\nu} - \delta_{\rho}^{\nu} T_{\theta}^{\theta\mu}\right). \quad (7)$$

From equations (2) - (4) the description of torsion scalar is specified by

$$T = S_{\rho}^{\ \mu\nu} T^{\ \rho}_{\ \mu\nu}\ . \quad (8)$$

The field equations of *F(T)* theory of gravity towards the action (4) reads as

$$e^{-1}\partial_{\mu}(e s_i^{\mu\nu} - e_i^{\lambda} T_{\mu\lambda}^{\rho}\, s_{\rho}^{\nu\mu}) F_T + s_i^{\mu\nu} \partial_{\mu} T\, F_{TT} + \frac{1}{4} e_i^{\nu} F = \frac{1}{2} k^2 e_i^{\rho}\, T_{\mu}^{\nu}, \quad (9)$$

Where$F_T = \frac{dF}{dT}$.

The energy momentum tensor which includes relationship along with the matter fields is defined as

$$T_{\mu}^{\nu} = diag\ (\rho, -p, -p, -p)\ , \quad (10)$$

here ρ and $p$represents energy density and pressure of the matter.

III. The Field Equations

A flat homogeneous and isotropic FLRW metric described the line element as

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \quad (11)$$

Where $a(t)$ denotes the scale factor and $t$represents the cosmic time.

For the metric (11), we have

$$e_{\mu}^{i} = diag\big(1, a(t), a(t), a(t)\big),$$

$$e_i^{\mu} = diag\big(1, a^{-1}(t),\ a^{-1}(t), a^{-1}(t)\big). \quad (12)$$

The expansion rate is expressed with reference to the scale factor by$H = \frac{\dot{a}}{a}$ .The function H is called Hubble parameter.

By using equations (5) – (7), the torsion tensor in equation (8) has the form

$$T = -6H^2\ . \quad (13)$$

By using equations (11) – (13) in equation (9), the modified Friedmann equations are obtained as

$$12H^2 F_T + F = 2k^2\rho\ , \quad (14)$$

$48H^2\dot{H}F_{TT} - (12H^2 + 4\dot{H})F_T - F = 2k^2p$.(15)

The conservation equation can be written as

$\dot{\rho} + 3H(\rho + p) = 0$. (16)

we can rewrite equation (14) and equation (15) as

$-2TF_T + F = 2k^2\rho$, (17)

$-8\dot{H}TF_{TT} + (2T - 4\dot{H})F_T - F = 2k^2p$. (18)

The conservation equation (16) can be written as

$\dot{\rho} + 3H(\rho + p) = 0$. (19)

The equation (19) can be written as

$\frac{d}{dt}(a^3\rho) = -3a^3Hp.$ (20)

Also, as *F(T)*= T equations (17) and (18) transform to the normal Friedmann equation of GR

$\frac{3}{k^2}H^2 = \rho , \frac{1}{k^2}\left(2H \dot{+} 3H^2\right) = -p.$ (21)

we can rewrite equation (17) and (18) as

$3\frac{H^2}{K^2} = \rho + \rho_T$, $\frac{1}{k^2}\left(2H^{\dot{}} + 3H^2\right) = -(p + p_T)$, (22)

where,

$\rho_T = \frac{1}{2k^2}(2TF_T - F + 6H^2),$ (23)

and

$p_T = -\frac{1}{2k^2}\left[4\dot{H} - F - 8\dot{H}TF_{TT} + 2TF_T - 4\dot{H}F_T + 6H^2\right]$. (24)

The above equation gives the presentation of torsion towards the energy density(ρ) and pressure(p).

The EoS parameter ω $= \frac{p}{\rho}$ has different values for three special cases.

From equations (14) and (15), the EoS parameter ω takes the form

$\omega = -1 + \frac{-8\dot{H}TF_{TT} - 4\dot{H}F_T}{-2TF_T + F}$. (25)

above equation (25) takes the form

$[(1 + \omega)2T - 4\dot{H}]F_T - 8\dot{H}TF_{TT} - (1 + \omega)F = 0$. (26)

IV. Construction of some *F(T)* Models using EoS Parameter

Now we construct *F(T)* for a particular value of the parameter of state ω.

Consider equation (25) in the following form

$(1 + \omega)(-\psi F_T + F) = \varphi\psi F_{TT} - \varphi F_T$. (27)

where$4\dot{H} = \varphi$ and 2T = ψ.

By using non-identical values of EoS parameter(ω) in equation (27), we consider the following cases and construct different *F(T)*models.

Case 1

For EoS parameter ω = 0, equation (27) gives

-φ $\psi F_{TT} + \varphi F_T - \psi F_T + F = 0.$ (28)

The above equation gives the following general solution

F(T)= $c_1e^{-\varphi(t)} + c_2e^{\psi(t)}$. (29)

where$c_1$and c_2 are arbitrary constants.

Case 2

In this case we take ω = -1, equation (27) takes the form

$\varphi\psi F_{TT} - \varphi F_T = 0.$ (30)

which gives the general solution in the form

F(T)= $c_3 + c_4e^{\frac{1}{\psi(t)}}$. (31)

wherec_3 and c_4 are arbitrary constants.

Case 3

Now for ω = $\frac{1}{3}$, equation (27) gives

$-\frac{4}{3}\psi F_T + \varphi F_T - \varphi\psi F_{TT} + \frac{4}{3}F = 0.$ (32)

which gives the following general solution as

F(T)= $c_5e^{\frac{4\psi t}{3}} + c_6e^{-\varphi(t)}$. (33)

wherec_5 and c_6 are arbitrary constants.

V. *F(T)*is given

Now let us consider that *F(T)* is given. From this the analogous expression for ρ(energy density) and p (pressure) are as follows.

***F(T)*= $\boldsymbol{\alpha_1T + \alpha_2T^{\beta}logT}$**

For the fundamental form we assume the succeeding *F(T)* model

F(T)= $\alpha_1T + \alpha_2T^{\beta}logT$. (34)

Then the modified Friedmann equation from (17) gives

$$\alpha_1 T + 2\alpha_2 T^\beta + \alpha_2 T^\beta \log T(2\beta - 1) = -2k^2\rho, \quad (35)$$

Also, from (18) we get,

$$\alpha_2(2\beta-1)T^{\beta-1}(T-4\beta\dot{H})\log T - 4\alpha_2(4\beta-1)\dot{H}T^{\beta-1} - 4\alpha_1\dot{H} + 2\alpha_2 T^\beta + \alpha_1 T = 2k^2 p. \quad (36)$$

To get simplest formulations of pressure (p) as well as of energy density(ρ)we take β = 0.5

we get

$$\alpha_1 T + 2\alpha_2 T^{0.5} = -2k^2\rho, \quad (37)$$

$$\alpha_1 T - 4\alpha_1 \dot{H} - 4\alpha_2 \dot{H}T^{-0.5} + 2\alpha_2 T^{0.5} = 2k^2 p, \quad (38)$$

respectively. Now we take $\alpha_1 = 0$, equation (37) and (38) takes the form

$$2\alpha_2 T^{0.5} = -2k^2\rho, \quad (39)$$

$$2\alpha_2 T^{0.5} - 4\alpha_2 \dot{H}T^{-0.5} = 2k^2 p, \quad (40)$$

The corresponding EoS parameter reads as

$$\omega = -1 + 2\dot{H}T^{-1}. \quad (41)$$

F(T)= $\boldsymbol{\alpha_1 T + \frac{\alpha_2}{T}}$

Now we consider following *F(T)* model

$$F(T) = \alpha_1 T + \frac{\alpha_2}{T}. \quad (42)$$

for this case we get equations (17) and (18) as

$$3\frac{\alpha_2}{T} - \alpha_1 T = 2k^2\rho, \quad (43)$$

$$-3\frac{\alpha_2}{T} + \alpha_1 T - 12\alpha_2\frac{\dot{H}}{T^2} - 4\alpha_1\dot{H} = 2k^2\rho, \quad (44)$$

The corresponding parameter of state reads as

$$\omega = -1 + \frac{4\dot{H}\,(3\alpha_2 T - \alpha_1 T)}{\alpha_1 T^2 - 3\alpha_2}. \quad (45)$$

F(T) = $\boldsymbol{\alpha_1 T + \alpha_2 T^n}$

Now *F(T)* has following form of model

$$F(T) = \alpha_1 T + \alpha_2 T^n. \quad (46)$$

We get the modified Friedmann equations as

$$-\alpha_1 T + \alpha_2 T^n(1-2n) = 2k^2\rho. \quad (47)$$

$$\alpha_2(1-n)T^n + \alpha_1 T - T^n\alpha_2(1-2n) - 4\alpha_2 n(2n-1)\dot{H}T^{n-1} - 4\alpha_1\dot{H} = 2k^2 p. \quad (48)$$

The expression for EoS parameter reads as

$$\omega = -1 - \frac{[\alpha_2(1-n)T^n - 4\alpha_2 n(2n-1)\dot{H}T^{n-1} - 4\alpha_1\dot{H}]}{\alpha_1 T - \alpha_2(1-2n)T^n}. \quad (49)$$

F(T) = λ T

Here we assume following *F(T)*model

$$F(T) = \lambda T. \quad (50)$$

for above model we get equation (17) and (18) as follows

$$-\lambda T = 2k^2\rho. \quad (51)$$

$$\lambda(T - 4\dot{H}) = 2k^2 p. \quad (52)$$

Then the resulting EoS parameter will be as

$$\omega = 4\frac{\dot{H}}{T} - 1. \quad (53)$$

VI. Conclusion and Discussion

In this article, we have examined a flat homogeneous and isotropic FLRW universe in *F(T)*theory of gravity. By making use EoS parameter we constructed *F(T)*for three different phases such as matter, radiation and DE. Also, we have constructed four cosmological models by assuming suitable function for *F(T)* as *F(T)*= $\alpha_1 T + \alpha_2 T^\beta logT$,*F(T)*= $\alpha_1 T + \frac{\alpha_2}{T}$, *F(T)* = $\alpha_1 T + \alpha_2 T^n$, *F(T)*= λ T. The physical properties studied here will be useful to understand the initial phases of universe. Cosmic evolution obtained from above models constitutes unification of inflation with late time acceleration derive from astronomical observations.

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