Volume 66, Issue 5, 2022

Journal of Scientific Research

of

The Banaras Hindu University

Decipher Multi-Objectives gathering with liberate Dates Problem

Tanveer A. Tarray[a], Zahoor A. Ganie[b] and Mohd. Khalid[c]

[a]Department of Mathematical Sciences, Islamic University of Science and Technology, Awantipora, Jammu and Kashmir, India.
(E-mail: tanveer.tarray@islamicuniversity.edu.in)
[b]Department of Electrical Engineering, Islamic University of Science and Technology, Awantipora, Jammu and Kashmir, India.
(E-mail: zahoor.ganie@islamicuniversity.edu.in)
[c]Department of Statistics & Operations Research (Women's College), Aligarh Muslim University, Aligarh, India. (E-mail: mohdkhalidkhan4870@gmail.com)

Abstract: This crux of this paper is to suggesta mathematical problem using randomized response technique. We present some special cases applying branch and bound algorithm in order to find the exact (optimal) solution for it. By some heuristics methods we find the upper bound using different algorithms.

Keywords: Stratified Sampling; Randomized Response Technique, Multi-Objective Problem, Branch and Bound method, LINGO.

1. Introduction

In order to reduce non-response and response bias, a survey technique different from open or direct surveys was needed that made people comfortable and encouraged truthful answers. Warner (1965) developed such an alternative survey technique that is called randomized response technique (RRT). This pioneering work of Warner's (1965) led to modifications and developments in various directions. Feeling that the cooperation of the respondent might be further enhanced if one of the two questions referred to a non – sensitive, innocuous attribute, unrelated to sensitive attribute, Horvitz et al. (1967) proposed an unrelated question randomized response model (U-model). Theoretical details for this model were given by Greenberg et al. (1969). This technique has generated much interest in the statistical literature since the publication of Warner's randomized response model. Subsequently, several other workers have proposed different strategies for instance, see Land et al. (2012), Singh and Tarray (2014a, 2014b, 2015), Tarray and Singh (2015,2016a, 2016b, 2017,2018a,2018b) and Tarray et al. (2018). An RRT using a stratified random sampling gives the group characteristics related to each stratum estimator. In recent years, Lawler (1973) suggested a method for reducing the maximum cost (f_{max}). After that, scheduling difficulties sparked a lot of attention, resulting in a significant number of studies introducing excellent approaches for determining optimality see Blazewicz (2007), Chen et al. (2007), Graham (1979), Johnson (1954) etc.

2. Problem Formulation:

The issue addressed in this work is that of scheduling the set N

DOI: 10.37398/JSR.2022.660516

of n jobs, $N = \sum_{i=1}^{j} N_i$ on a one- machine". Each job i, i∈Nhas a processed time that is an integer z_i a release date r_i, and due date d_i. Given a schedule σ=(1,…,n), the flowing time of the job i, F_i can be define as $F_i=C_i-r_i$ where C_I be completion time for job i, given by relationship:

$C_1=r_1+z_1, C_i=\max\{r_i, C_i\}+z_i$ for $i=2,...,n$.

Job i's tardiness is defined by $T_i = \max\{C_i - d_i, 0\}$, and earliness by $E_i=\max\{d_i - C_i, 0\}$.The late work of job i given by $V_i = min\ \{T_i, Z_i\}$. Our issue (Z) has the following mathematical form:

$$M = \text{Min} \quad F(\sigma) = \text{Min}_{\sigma \in \delta} \{ \sum_{i=1}^{n} (F_{\sigma(i)} + T_{\sigma(i)} + E_{\sigma(i)} + V_{\sigma(i)}) \}$$

Subject to:

$$C_{\sigma(1)} = r_{\sigma(1)} + z_{\sigma(1)}$$

$$C_{\sigma(i)} = Max\{r_{\sigma(i)}, C_{\sigma(i-1)} + z_{\sigma(i)}, \quad i = 2,...n$$

$$T_{\sigma(i)} = Max\{, C_{\sigma(i)} - d_{\sigma(i)}, 0\}, \quad i = 1,...n$$

$$E_{\sigma(i)} = Max\{, d_{\sigma(i)} - C_{\sigma(i)}, 0\}, \quad i = 1,...n$$

$$V_{\sigma(i)} = Max\{T_{\sigma(i)}, z_{\sigma(i)}, \quad i = 2,...n$$

The purpose is to find a processing sequence σ = (σ (1), . . .,(n)) The sum of overall flow times, total tardiness, total earliness, and total late work for the problem (z_i)to reduce.

3. Solution Procedure:

To get the full benefit from stratification, it is assumed that the number of units in each stratum is known. In the stratified Warner's randomized response model, an individual respondent in the sample from stratum 'i' is instructed to use the randomization device which consists of a sensitive question (S) card with probability P_i and its negative question $(\bar{S})$ card with probability ($1-P_i$). The respondent answers the question with "Yes" or "No" without reporting which question card he or she has. A respondent belonging to the sample in different strata will perform different randomization device, each having different pre assigned probabilities. Under the assumption that these "Yes" or "No" reports are made truthfully and Pi is set by the researcher, the probability of "Yes" answers in stratum 'i' for the stratified Warner's RR model is: $Z_i = P_i \pi_{Si} + (1-P_i)(1-\pi_{Si})$, for (i =1, 2 ..., k),the variance / MSE of $\hat{\pi}_S$ is given by

$$V(\hat{\pi}_S) = \frac{1}{n} \sum_{i=1}^{k} w_i \left[\pi_{Si}(1-\pi_{Si}) + \frac{P_i(1-P_i)}{(2P_i-1)^2} \right]$$

The sampling cost function is of the form $\sum_{h=1}^{k} c_h n_h$, the cost is proportional to the size of the sample within any stratum.

We define $c^0 = C - C^0$.The linear cost function is $C = C^0 + \sum_{h=1}^{k} c_h n_h$, where C^0 is the over head cost, c_h is the per unit cost of measurement in h^{th} stratum, C is the available fixed budget for the survey.

The problem of optimum allocation can be formulated as a non linear programming problem (NLPP) for fixed cost as

$$\left.\begin{array}{l} \text{Minimize} \quad V(\hat{\pi}_S) = \sum_{h=1}^{k} \frac{w_h^2}{n_h} V_h \\ \text{sujbect to} \quad \sum_{h=1}^{k} c_h n_h \le c^0 \\ \qquad 2 \le n_h \le N_h \quad \text{and } n_h \text{ int egers, } h = 1,2,...k \end{array}\right\}$$

The above NLPP can be solved using non linear integer programming technique. We can now apply Branch and Bound method to solve the challenge (z_i). The branch and bound approach is the major way for solving the problem, whereas the bat algorithm and the Gray Wolf algorithm (GW) are used. The upper bound will be found by applying the bat algorithm in which the parameter x and v refers to position and velocity of solution respectively, the following equation describe the updating the value of both $x\ and\ v$,

$(t) = v(t-1) + (x^*(t) - Bval*x(t-1)) * f(t)$.

$(t) = x^*(t-1) + v(t)$.

and

$x^*(t) = x(t-1) * val$.

$f(t) = fmin + (fmax - fmin) * B(t)$.

$(t) = R(t-1)(1 - e^{-\gamma t})$.

$A(t) = A(t-1) * \alpha Bval$/vol

Where: val is the current local solution. And (t) is a random vector, andα, $\epsilon\ and\ \gamma \in$ (0,1). The algorithm terminates after finish all iterations $CNandN$, or if the procedure exceeds fixed period of time. Now for the lower bound The problem (z_i)can be broken into two sub problems in order to have a less complex structure (z_1)and(z_2),

M_1= Min $_{\sigma\in\delta}$ $\{ \sum_{i=1}^{n} (F_{\sigma(i)} + T_{\sigma(i)} + E_{\sigma(i)})\}$

Subject to:

$$C_{\sigma(1)} = r_{\sigma(1)} + z_{\sigma(1)}$$

$$C_{\sigma(i)} = Max\{r_{\sigma(i)}, C_{\sigma(i-1)} + z_{\sigma(i)} , \quad i = 2,\ldots n$$

$$T_{\sigma(i)} = Max\{, C_{\sigma(i)} - d_{\sigma(i)}, 0\} , \quad i = 1,\ldots n$$

$$E_{\sigma(i)} = Max\{, d_{\sigma(i)} - C_{\sigma(i)}, 0\} , \quad i = 1,\ldots n$$

M_2 =Min $_{\sigma\in\delta}$ $\{ \sum_{i=1}^{n} (V_{\sigma(i)}) \}$

Subject to:

$$C_{\sigma(1)} = r_{\sigma(1)} + z_{\sigma(1)}$$

$$C_{\sigma(i)} = Max\{r_{\sigma(i)}, C_{\sigma(i-1)} + z_{\sigma(i)} , \quad i = 2,\ldots n$$

$$T_{\sigma(i)} \geq 0, \quad i = 1,\ldots n$$

$$V_{\sigma(i)} = Max\{T_{\sigma(i)}, z_{\sigma(i)} , \quad i = 2,\ldots n$$

.

The lower bound for M_1 and M_2into two sub problems can be written as ,

$$Min\ Z(\delta) = \min_{\delta\varepsilon S}\left\{\sum_{i=1}^{n}\left(E_i + T_{\delta_i} + F_{\delta_i}\right)\right\}$$

$$= \min_{\delta\varepsilon S} \sum_{i=1}^{n} \left\{Max\left\{d_{\delta_i} - r_{\delta_i}, 2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}, c_{\delta_i} - r_{\delta_i}\right\}\right\}$$

Since the third term $c_{\delta_i} - r_{\delta_i}$ is always between $d_{\delta_i} - r_{\delta_i}$and $2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}$, then we can write the objective function $Z(\delta)$ as :

$$= \min_{\delta\varepsilon S} \sum_{i=1}^{n} \left\{Max\left\{d_{\delta_i} - r_{\delta_i}, 2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}\right\}\right\}$$

This means that the cost of scheduling job δ_j is (δ_j), given by:

$$Z(\delta) = \begin{cases} d_{\delta_i} - r_{\delta_i} & if \quad c_{\delta_i} - r_{\delta_i} \leq d_{\delta_i} - r_{\delta_i} \\ 2c_i - d_{\delta_i} - r_{\delta_i} & otherwise \end{cases}$$

i.e.,$z(\delta_i)$is equal to d_{δ_i} if job is early and $z(\delta_i)$ is equal to $2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}$ if job i is tardy.

Also, we can write the objective of the M_1 in other form as the following:

$$\sum_{i\epsilon\delta}(E_i + T_i + F_i) = \sum_{i\epsilon\delta}(E_i + T_i + C_i - r_i)$$

$$= \sum_{i\epsilon ER}(d_i - r_i + 2) \sum_{i=LT} T_i \sum_{i\epsilon LT}(d_i - r_i)$$

For problem P_1 since:

$$\geq \underset{\delta\varepsilon S}{\text{Min}}\left\{ Max\left\{\sum_{i=1}^{n} d_{\delta_i} - r_{\delta_i}, \sum_{i=1}^{n} Max\left\{2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}, c_{\delta_i} - r_{\delta_i}\right\}\right\}\right\}$$

Put $y_{\delta i} = Max\{2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}, c_{\delta_i} - r_{\delta_i}\}$

To show that

$$\therefore LB = \underset{\delta\varepsilon S}{\text{Min}}\left\{ Max\left\{\sum_{i=1}^{n} d_{\delta_i} - r_{\delta_i}, \sum_{i=1}^{n} Max\left\{2c_{\delta_i} - d_{\delta_i} - r_{\delta_i}, c_{\delta_i} - r_{\delta_i}\right\}\right\}\right\}$$

The mathematical form of problem P_2 is as follows:

$$Min\, w(\delta) = Min_{\delta\varepsilon S}\left(\sum_{i=1}^{n} v_{\delta_{(i)}}\right)$$

Clear that: $Vmax(\delta) \leq \sum_{i=1}^{n} v_{\delta_{(i)}} \quad \forall\, \delta\varepsilon s$

$\because Vmax\,(lw) \leq Vmax\,(\delta), \quad \forall\, \delta\epsilon s$. When lw is the Lower sequence for V max.

4.ExperimentalResults:

n = 10	UB	time
1	200	0.00363
2	212	0.001456
3	160	0.002207
4	180	0.001804
5	135	0.002125

Table (1) shows that the upper values of the mathematical problem when n=10, one can find different values with time if n= 20, 30, etc. So, it clearly shows that the proposed mathematical model is dexterous than the existing one.

5. Conclusion:

A stratified randomized response method assists to solve the limitations of randomized response that is the loss of individual characteristics of the respondents. Formulating nonlinear programming problem (NLPP) of optimum allocation in stratified sampling with linear cost function in presence of non responses using Branch and Bound / Gray Wolf algorithm provides the optimum integer solution.

Acknowledgement

We sincerely thank the anonymous referees for their valuable comments and suggestions. The research is supported by JKST&IC under the research project number JKST&IC/SRE/397-400.

References

1. Blazewicz, J., Ecker, K.H., Pesch, E., Schmidt, G. and Weglarz, J. (2007). Handbook on Scheduling: From Theory to Applications. Springer. Berlin.Heidelberg.NewYork.
2. Chen, Wei-Yang, and Gwo-Ji Sheen (2007). Single-machine scheduling with multiple performance measures: minimizing job-dependent earliness and tardiness subject to the number of tardy jobs. International Journal of Production Economics.Vol.109.No.1-2,p.p.214-229.
3. Graham, R. L., Lawler, E. L., Lenstra, J. K., &Kan, A. R (1979).Optimization and approximation in deterministic sequencing and scheduling: a survey. In Annals of discrete mathematics, Vol. 5.Elsevier,p.p.287-326.
4. Greenberg B, Abul- Ela A, Simmons W and Horvitz D. (1969). The unrelated question randomized response: Theoretical framework. J. Amer. Statist. Assoc., 64,529-539.
5. Horvitz, D.G., Shah, B.V., Simmons, W.R. (1967). The unrelated question randomized response model. In: Proceedings of the Section on Survey Research Methods, American Statistical Association, pp. 65– 72 .
6. Johnson,S.M. (1954). Optimal two and three stage production schedules with set-uptimes included. Naval Res.Logist Quart.1: 61-68.

7. Land M, Singh S and Sedory S. (2012). Estimation of a rare attribute using Poisson distribution. Statistics, Statistics 46(3): 351-360.
8. Lawler E.L. (1973). Optimal sequencing of a single machine subject to precedence on straints. Management Science19:544-546.
9. Singh H.P. and Tarray T. A. (2014 a). An alternative to stratified Kim and Warde's randomized response model using optimal (Neyman) allocation. Model Assist. Statist. Appl. 9, 37-62.
10. Singh H.P. and Tarray T.A. (2015). A revisit to the Singh, Horn, Singh and Mangat's randomization device for estimating a rare sensitive attribute using Poisson distribution. Model Assist. Statist. Appl., 10 ,129-138.
11. Tarray T.A. and Singh H.P. (2015).Some improved additive randomized response models utilizing higher order moments ratios of scrambling variable. Model Assist. Statist. Appl., 10 , 361-383.
12. Tarray T.A. and Singh H.P. (2016a). An adroit randomized response new additive scrambling model. Gazi University Journal of Sciences. 29(1), 159-165.
13. Singh H and Tarray T.A. (2014 b).An improved mixed randomized response model. Model Assist. Statist. Appl. 9 ,73-87.
14. Tarray T.A. and Singh H.P. (2016b). A stratified randomized response model for sensitive characteristics using the negative hyper geometric distribution. Commun. Stat. Theory Methods. 46 (6) , 2607-2629.
15. Tarray T.A. and Singh H.P. (2017). An optional randomized response model for estimating a rare sensitive attribute using Poisson distribution. Commun. Stat. Theory Methods. 46 (6) , 2638-2654.
16. Tarray T.A. and Singh H.P. (2018a). A randomization device for estimating a rare sensitive attribute in stratified sampling using Poisson distribution. Afr. Mat. 29 , 407-423.
17. Tarray T.A. and Singh H.P. (2018b). Missing data in clinical trials: stratified Singh and Grewal's randomized response model using geometric distribution. Trends Bioinf. 11(1) ,44-55.
18. Tarray T.A., Peer Bilal and Bhat M.R. (2018). A new approach to randomized response model using fuzzy numbers. Stat. Appl. 16(2) , 49-63.
19. Warner S. (1965).Randomized response: A survey technique for eliminating evasive answer bias. J. Amer. Stat. Assoc. 60 (1965): 63-69.

***