# A study on multipole mixing ratio of $\gamma$-transitions from unfavoured to favoured signature partner of the $\pi h_{11 / 2}$ band in ${ }^{129} \mathrm{Cs}$ 

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#### Abstract

Angular distribution coefficients, $a_{2}$ and $a_{4}$, of the $\Delta I=1 \gamma$-transitions between the $\alpha= \pm 1 / 2$ signature partners of $\pi h_{11 / 2}$ band in ${ }^{129} \mathrm{Cs}$ have been estimated for different multipole mixing ratio ( $\delta$ ) and compared with their experimental values available in the literature. From this study, a considerable amount of the $E 2$ admixture is found in these transitions, especially at higher angular momentum. This provides further evidence for tri-axial nuclear shapes in ${ }^{129} \mathrm{Cs}$.


Index Terms—Angular distribution, Multipole mixing ratio.

## I. Introduction

Precise measurement of the multipole mixing ratio ( $\delta$ ) of $\Delta I=1 \gamma$-transitions become an important topic of research nowadays in the context of the low lying collective excitations in triaxial nuclei which have three unequal principal axes of the nuclear density distribution. Such nuclei can rotate collectively about any of these three axes with different moments of inertia. The wobbling motion is uniquely related to the non-axial shape of an atomic nucleus and hence is considered as a fingerprint of nuclear triaxiality (Hagemann, 2005). Existence of wobbling mode gives rise to a family of bands with similar rotational properties like dynamical moments of inertia, quasiparticle alignments etc. But, these rotational properties are also found similar in the case of signature partner bands in deformed nuclei (Jónsson et al., 1984). Thus, the conclusive evidence of the wobbling motion comes from the electromagnetic properties of the connecting $\Delta I=1 \gamma$-decays between two successive phonon wobbling bands. While $\Delta I=1$ transitions between signature partners are found primarily $M 1$ in nature (Hamamoto and Sagawa, 1979; Hagemann and Hamamoto, 1989), the $\Delta I=1 \gamma$-rays between consecutive wobbling bands have predominately $E 2$ character due to the involvement of the entire nuclear charge (Bohr and Mottelson, 1975). This decomposition in $E 2$ and $M 1$ amplitudes can be determined from the multipole mixing ratio.

Recent studies on transitional nuclei in $A \approx 130$ region opens up a new horizon for research on wobbling motion
in atomic nuclei. So-called 'unfavoured signature partner' of $\pi h_{11 / 2}$ [ $\nu h_{11 / 2}$ ] band, after some addition and/or alteration, in ${ }^{135} \mathrm{Pr}$ (Matta et al., 2015) and ${ }^{133} \mathrm{La}$ (Biswas et al., 2019) [ ${ }^{127} \mathrm{Xe}$ (Chakraborty et al., 2020) and ${ }^{133} \mathrm{Ba}$ (Rojeeta Devi et al., 2021)] is re-interpreted in terms of wobbling excitation based on the large $\delta$ value of the connecting $\gamma$-transitions. However, the wobbling interpretation of some of these lowlying bands is questioned severely in the light of recent experimental as well as theoretical results (Lv et al., 2022). Thus, it becomes important to revisit the available experimental data in this mass region. In this work, the ${ }^{129} \mathrm{Cs}$ nucleus has been chosen as a test case. The available experimental angular distribution results have been compared with their theoretical estimates to determine the multipole mixing ratio.

## II. Literature Survey

Low spin states in ${ }^{129}$ Cs were initially identified from the $\beta$-decay of ${ }^{129} \mathrm{Ba}$. Taylor and co-workers identified about 100 new $\gamma$-rays using $\mathrm{Ge}(\mathrm{Li})$ and scintillation detectors and developed a decay scheme of ${ }^{129} \mathrm{Cs}$ upto $E_{x} \approx 2.4 \mathrm{MeV}$ (Taylor et al., 1972). They have also assigned the spins and parities of few excited states upto $E_{x} \approx 221 \mathrm{keV}$ on the basis of the $\beta$-decay characteristics and the multipolarities, determined from the conversion electron and $\gamma$-ray intensities, of the low-energy $\gamma$-rays. However, the lowest negative parity state at $E_{x}=575.6 \mathrm{keV}$ was first identified by Ishii et al. from the observation of a $E_{\gamma}=149 \mathrm{keV}$ transition in coincidence with $E_{\gamma}=420 \mathrm{keV} \gamma$-ray and $E_{\gamma}=387 \mathrm{keV}$ transition in coincidence with $E_{\gamma}=182 \mathrm{keV} \gamma$-ray from the $\beta$-decay of ${ }^{129} \mathrm{Ba}$ (Ishii et al., 1973).

A few higher spin negative parity states were first identified by J. Chiba and co-workers from an in-beam $\gamma$-ray spectroscopic study using coaxial $\mathrm{Ge}(\mathrm{Li})$ detectors, following the ${ }^{127} \mathrm{I}\left({ }^{4} \mathrm{He}, 2 \mathrm{n} \gamma\right){ }^{129} \mathrm{Cs}$ fusion-evaporation reaction at $E_{\text {beam }}=28$ MeV (Chiba et al., 1977). Spins and parities of these states were assigned on the basis of the multipolarities of the decay-
ing $\gamma$-rays, determined from the angular distribution measurement. They also measured the half-life, $\tau_{1 / 2}=0.69(3) \mu \mathrm{s}$, of the $I^{\pi}=11 / 2^{-}$state.

A plethora of systematic experimental works on odd- $A$ Cs isotopes was reported by U. Garg and co-workers using ${ }^{4} \mathrm{He}$, ${ }^{6} \mathrm{Li},{ }^{10} \mathrm{~B},{ }^{14} \mathrm{~N}$ and ${ }^{16} \mathrm{O}$ induced fusion-evaporation reactions (Garg et al., 1979a,b). They used large volume coaxial Ge(Li) detectors to detect the de-exciting $\gamma$-rays and measured $\gamma \gamma$ coincidence and $\gamma$-ray angular distribution measurements. But, the spectroscopic information on ${ }^{129} \mathrm{Cs}$, the nucleus of present interest, is somewhat limited (Garg et al., 1979a).

The level scheme of ${ }^{129} \mathrm{Cs}$ was extended significantly to the higher spin by L. Hildingsson and co-workers from heavyion induced ${ }^{122} \mathrm{Sn}\left({ }^{11} \mathrm{~B}, 4 \mathrm{n} \gamma\right)$ and ${ }^{116} \mathrm{Cd}\left({ }^{18} \mathrm{O}, \mathrm{p} 4 \mathrm{n} \gamma\right)$ fusionevaporation reactions (Hildingsson et al., 1991). The NORDBALL (Herskind, 1986) array of HPGe detectors was used to measured the coincidence relationship. On the other hand, the angular distribution of the $\gamma$-rays was measured using two Compton-suppressed HPGe detectors.

Further spectroscopic investigation on this nucleus was carried out, with an array of fourteen Compton-suppressed HPGe detectors, by Yan-Xin et al. following the ${ }^{122} \mathrm{Sn}\left({ }^{11} \mathrm{~B}\right.$, $4 \mathrm{n} \gamma)^{129} \mathrm{Cs}$ reaction (Yan-Xin et al., 2009). Apart from extending the previously known bands to higher spins, two additional bands were also placed in the level scheme and one of them was identified as the $\gamma$-vibrational band built on $\pi h_{11 / 2}$ orbital.

A major revision of the level scheme of ${ }^{129} \mathrm{Cs}$ was done by S. Sihotra and co-workers by adding about sixty new $\gamma$ rays (Sihotra et al., 2009). The high spin states were populated through ${ }^{122} \mathrm{Sn}\left({ }^{11} \mathrm{~B}, 4 \mathrm{n} \gamma\right){ }^{129} \mathrm{Cs}$ reaction. The Gamma Detector Array (Muralithar, 2014) having twelve Comptonsuppressed Ge detectors and a fourteen-element bismuth germanate (BGO) multiplicity filter was used to detect the valid $\gamma$-events. Spins and parities of several excited levels were confirmed / assigned on the basis of $\gamma \gamma$ angular correlations.

Apart from the aforementioned spectroscopic measure-


Fig. 1. The $\pi h_{11 / 2}$ band in ${ }^{129} \mathrm{Cs}$ (Sihotra et al., 2009).
ments, the lifetimes of few excited states in this nucleus were also measured via Doppler shift attenuation method (DSAM). Lie-Lin et al. measured the lifetimes of five negative parity and six positive parity states, belonging to the favoured signature partners of $\pi\left[h_{11 / 2}, g_{7 / 2}\right.$ and $\left.d_{5 / 2}\right]$ bands in ${ }^{129} \mathrm{Cs}$, with the help of an array of fourteen Compton-suppressed HPGe detectors (Lie-Lin et al., 2010). Recently, Lamani and co-workers measured the lifetimes of nine negative parity and eight positive parity states below and above the band-crossing utilising twenty-one Compton-suppressed clover HPGe detectors of Indian National Gamma Array (Palit et al., 2012; Lamani et al., 2021). Both of these studies were carried out following ${ }^{124} \mathrm{Sn}\left({ }^{11} \mathrm{~B}, 6 \mathrm{n} \gamma\right){ }^{129} \mathrm{Cs}$ fusion-evaporation reaction.

## III. Results \& Discussions

## Angular Distribution of the $\gamma$-rays

In the heavy-ion induced nuclear reactions, the incoming projectile brings in a large linear momentum along the beam axis and it leads to a large angular momentum aligned in the plane perpendicular to the beam axis. Consequently, strongly oriented compound nuclei, with respect to the beam axis, are formed. Emitted $\gamma$-rays from such aligned states show characteristic angular distributions depending on their multipolarities and the angular momentum of the involved states. Thus, the angular distribution measurement becomes very useful to assign the spin of an excited nuclear state. The angular distribution for a $\gamma$-ray of multipole order ' $l$ ' is usually expressed as:

$$
\begin{equation*}
W_{l}(\theta)=\sum_{k=0}^{l} A_{2 k} P_{2 k}(\cos \theta) \tag{1}
\end{equation*}
$$

where, $A_{2 k}$ represents the angular distribution coefficients (alignment parameters), $P_{2 k}$ stands for the even numbered Legendre polynomials and $\theta$ is the angle of the detector w.r.t. the beam direction. In the heavy-ion induced nuclear reactions, the excited states in a nucleus are populated mostly close to the yrast line and as a consequence, the observed $\gamma$-transitions are mainly of $l=1,2$ multipole order (Ferguson, 1965; Regan, 2003; Taras and Haas, 1975). Therefore, the eqn. (1) can be expressed as:

$$
\begin{align*}
W(\theta) & =A_{0}+A_{2} P_{2}(\cos \theta)+A_{4} P_{4}(\cos \theta) \\
& =A_{0}\left[1+a_{2} P_{2}(\cos \theta)+a_{4} P_{4}(\cos \theta)\right] \tag{2}
\end{align*}
$$

where, $a_{2}=A_{2} / A_{0}, a_{4}=A_{4} / A_{0}$ and the magnitudes of $A_{0}$, $A_{2}$ and $A_{4}$ can be estimated by measuring the $\gamma$-intensities at different $\theta$.

## Estimation of theoretical angular distribution coefficients

The spin alignment of a residual nucleus, populated in a nuclear reaction, changes slightly due the emission of particles or $\gamma$-rays. The $A_{2 k}$ coefficients have maximum magnitudes for the $\gamma$-rays emitted from completely aligned states. For such cases, the eqn. (1) can be written as:

$$
\begin{equation*}
W(\theta)=\sum_{k} A_{2 k}^{\max } P_{2 k}(\cos \theta) \tag{3}
\end{equation*}
$$



Fig. 2. Contour plots of the calculated angular distribution coefficients, $a_{2}$ and $\mathrm{a}_{4}$, for different mixing ratios $\delta$ (black line). The corresponding dispersion of experimental data, as per Ref. (Hildingsson et al., 1991), is marked with red crosses. The $\chi^{2}$ analysis for experimental distribution of corresponding $\gamma$-rays is shown in the inset (blue).

For the case of partial alignment, the angular distribution of a $\gamma$-transition can be expressed as:

$$
\begin{equation*}
W(\theta)=\sum_{k} \alpha_{2 k} A_{2 k}^{\max } P_{2 k}(\cos \theta) \tag{4}
\end{equation*}
$$

where, $\alpha_{2 k}=A_{2 k}^{e x p} / A_{2 k}^{\max }$ are the attenuation coefficients.
The $A_{2 k}^{\max }$ anisotropy coefficients are defined as:

$$
\begin{align*}
A_{2 k}^{\max }\left(J_{i} L_{1} L_{2} J_{f}\right)= & \frac{1}{1+\delta^{2}}\left[f_{2 k}\left(J_{f} L_{1} L_{1} J_{i}\right)\right. \\
& +2 \delta f_{2 k}\left(J_{f} L_{1} L_{2} J_{i}\right)  \tag{5}\\
& \left.+\delta^{2} f_{2 k}\left(J_{f} L_{2} L_{2} J_{i}\right)\right]
\end{align*}
$$

where,

$$
\begin{equation*}
\delta \equiv \frac{\left\langle J_{f}\left\|L_{2}\right\| J_{i}\right\rangle}{\left\langle J_{f}\left\|L_{1}\right\| J_{i}\right\rangle} \tag{6}
\end{equation*}
$$

is the multipole mixing ratio.
The quantity $f_{2 k}(J L)$ is expressed as:

$$
\begin{equation*}
f_{2 k}\left(J_{f} L_{1} L_{2} J_{i}\right) \equiv B_{2 k}\left(J_{i}\right) F_{2 k}\left(J_{f} L_{1} L_{2} J_{i}\right) \tag{7}
\end{equation*}
$$

where, $B_{2 k}(J)$ is the statistical tensor for complete alignment.
The $F_{2 k}(J L)$ term can be expressed as:

$$
\begin{align*}
F_{2 k}\left(J_{f} L_{1} L_{2} J_{i}\right) \equiv & (-1)^{J_{f}-J_{i}-1} \\
& \times \sqrt{\left(2 L_{1}+1\right)\left(2 L_{2}+1\right)\left(2 J_{i}+1\right)}  \tag{8}\\
& \times\left\langle L_{1} 1 L_{2}-1 \mid k 0\right\rangle \\
& \times W\left(J_{i} J_{i} L_{1} L_{2} ; k J_{f}\right)
\end{align*}
$$

where, $\left\langle L_{1} 1 L_{2}-1 \mid k 0\right\rangle$ is a Clebsch-Gordan coefficient and $W\left(J_{i} J_{i} L_{1} L_{2} ; k J_{f}\right)$ is a Racah coefficient.

For the $\gamma$-transitions of present interest in ${ }^{129} \mathrm{Cs}$, the angular distribution coefficients have been calculated for different

TABLE I
LIST OF EXPERIMENTAL ANGULAR DISTRIBUTION COEFFICIENTS,
REported in Ref. (Hildingsson et al., 1991), and EXtracted MULTIPOLE MIXING RATIO OF CORRESPONDING $\Delta I=1 \gamma$-RAYS IN ${ }^{129} \mathrm{CS}$.

| $E_{\gamma}$ | $I_{i}^{-} \rightarrow I_{f}^{-}$ | $a_{2}$ | $a_{4}$ | $\delta$ |
| :---: | :---: | :---: | :---: | :---: |
| 575 | $13 / 2 \rightarrow 11 / 2$ | $-0.76(12)$ | $+0.05(11)$ | -0.4 or -1.6 |
| 668 | $17 / 2 \rightarrow 15 / 2$ | $-0.66(11)$ | $+0.08(10)$ | -0.3 or -2.2 |
| 692 | $21 / 2 \rightarrow 19 / 2$ | $-0.39(10)$ | $+0.23(10)$ | -0.1 or -4.8 |
| 700 | $25 / 2 \rightarrow 23 / 2$ | $-0.65(18)$ | $+0.27(17)$ | -0.4 or -2.4 |

values of $\delta$ and shown as a contour of $a_{2}$ and $a_{4}$ in Fig. 2. The attenuation coefficients, $\alpha_{2}$ and $\alpha_{4}$, for $\sigma / J=0.3$ are taken from Ref. (Der Mateosian and Sunyar, 1974). Here, $\sigma$ denotes the half-width of Gaussian distribution of $m$ sub-states of $J$. The values $B_{2} F_{2}$ and $B_{4} F_{4}$ are adopted from Ref. (Yamazaki, 1967) for appropriate spin sequence.

## Experimental angular distribution coefficients

Experimentally, the angular distribution of $\gamma$-rays in ${ }^{129} \mathrm{Cs}$ were measured by Hildingsson et al., using ${ }^{122} \mathrm{Sn}\left({ }^{11} \mathrm{~B}\right.$, $4 \mathrm{n} \gamma$ ) ${ }^{129}$ Cs reaction at $\mathrm{E}_{\text {beam }}=50 \mathrm{MeV}$ (Hildingsson et al., 1991). Two Compton-suppressed HPGe detectors were used to measure the intensities of the $\gamma$-rays at different angles $(\theta)$ with respect to the beam direction. One out of these two detectors was used as the monitor and placed at $\theta=-90^{\circ}$. The other one was employed to measure the intensities of the $\gamma$-rays at $\theta=90^{\circ}, 115^{\circ}, 125^{\circ}, 135^{\circ}$. The $\gamma$-intensities were fitted to the standard Legendre polynomial [eqn. (2)] to get the angular distribution coefficients, which are listed in TABLE I.

## Determination of multipole mixing ratio: The $\chi^{2}$ analysis

The experimental angular distribution coefficients were compared with the theoretically estimated values of these coefficients (Fig. 2) to determine the multipole mixing ratio, $\delta$. The $\chi^{2}$ is calculated for different values of $\delta$ and the minimum of $\chi^{2}$ is indicative of the most probable magnitude of $\delta$, as shown in the inset of Fig. 2. The $\chi^{2}(\delta)$ is defined as:

$$
\begin{equation*}
\chi^{2}=\sum_{i}\left|\frac{W_{\text {theo }}\left(\theta_{i} J_{i} J_{f} \delta\right)-W_{\text {expt }}\left(\theta_{i}\right)}{\Delta W_{\text {expt }}\left(\theta_{i}\right)}\right|^{2} \tag{9}
\end{equation*}
$$

where, $W_{\text {theo }}$ is the theoretical angular distribution at an angle $\theta_{i}$ and $W_{\text {expt }}$ is the experimental distribution with a standard deviation $\Delta W_{\text {expt }}$. The uncertainty in $\delta$ can be considered at $1 \%$ confidence limit (Taras and Haas, 1975). Alternatively, one may follow the prescription given by James et al. to estimate the uncertainty present in $\delta$ (James et al., 1974).


Fig. 3. Percentage $E 2$ fraction as a function of mixing ratio $(\delta)$.


Fig. 4. Variation of the theoretical DCO ratio as a function of the mixing ratio ( $\delta$ ) for 668 keV and $692 \mathrm{keV} \gamma$-rays (black line). The solid (dashed) red lines indicate the experimental $\mathrm{R}_{\mathrm{DCO}}$ (uncertainty) of the corresponding $\gamma$-rays, as reported in Ref. (Sihotra et al., 2009).

From the present $\chi^{2}$ analysis, two possible values of $\delta$ $(|\delta|<0.5$ or $|\delta|>1.5)$ have been obtained with different probability (TABLE I). The $|\delta|<0.5$ corresponds to an $E 2$ fraction less than $20 \%$ (Fig. 3). On the other hand, the $E 2$ fraction is found as large as $70 \%$ or more in the case of $|\delta|>1.5$. It is evident from Fig. 2 that the lower magnitude of $\delta$ is more probable for $E_{\gamma}=575$ and 668 keV transitions in contrast to the cases of $E_{\gamma}=692$ and 700 keV transitions where the higher magnitude of $\delta$ is dominated. This makes the scenario more complicated as the lower magnitudes of $\delta$ support the signature relationship, but, the higher values of $\delta$ are indicative of the presence of some collective mode of excitations like wobbling. However, as the uncertainty in the measured $a_{4}$ coefficient is quite large, therefore, to understand such an electromagnetic behaviour of the $\Delta I=1 \gamma$-rays further experimental investigation is needed.

## Estimation of $\delta$ from the angular correlation measurement

The mixing ratio $\delta$ can also be determined by comparing experimentally measured $\mathrm{R}_{\mathrm{DCO}}$, the ratio for Directional Correlations from Oriented states (Krämer-Flecken et al., 1989), with its theoretical estimates. In the present work, theoretical $\mathrm{R}_{\mathrm{DCO}}$ of $17 / 2^{-} \rightarrow 15 / 2^{-}(668 \mathrm{keV})$ and $21 / 2^{-} \rightarrow 19 / 2^{-}$ ( 692 keV ) transitions have been calculated using computer code ANGCOR (Macias et al., 1976) and compared with earlier reported experimental $\mathrm{R}_{\mathrm{DCO}}$ (Sihotra et al., 2009) as shown in Fig. 4. It is evident from this figure that the experimental DCO ratio of both the $\gamma$-rays satisfy two values of $\delta_{\text {ang. corr. }}$, which are found in agreement with $\delta_{\text {ang. dist. within the limit }}$ of uncertainty. Unlike full angular distributions, however, the DCO ratios are not providing a unique value of $\delta$ for the $\gamma$-rays of present interest. Thus, one needs to combine the DCO data with some other spectroscopic results (e.g., linear polarization) to determine the correct $\delta$ value. Therefore, in addition to a more precise angular correlation measurements, the linear polarization of these $\gamma$-rays also needs to be measured in order to determine the mixing ratio unequivocally.

## IV. Summary

Electromagnetic character of the $\Delta I=1 \gamma$-decays between the signature partners of $\pi h_{11 / 2}$ band in ${ }^{129} \mathrm{Cs}$ has been studied in the light of angular distribution results. Extracted
magnitude of the $E 2 / M 1$ multipole mixing ratio, however, is not absolutely conclusive due to the presence of large uncertainty in the experimentally measured $a_{4}$ angular distribution coefficient. To decompose the $E 2$ and $M 1$ amplitudes of these $\Delta I=1 \gamma$-transitions, further experimental studies with precise angular distribution / correlation and linear polarization measurements are required. Thus, the present work sheds new light on the low-lying negative parity states in ${ }^{129} \mathrm{Cs}$, questioning their signature relationship and demands further experimental investigations.

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