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CONSTRUCTION AND ANALYSIS OF SPLIT-PLOT DESIGN USING SUDOKU SQUARE DESIGN STRUCTURE

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Abstract: *Split–plot design is an important statistical problem in the Design and Analysis of Experiments. These designs are widely used in experiments where one set of factors may require many experimental materials while another might be applied to smaller experimental materials. Sudoku square design consists of treatments that are arranged in a square array such that each row, column, or sub-square of the design contains each of the treatments only once. Therefore, this paper proposes the construction and analysis* ofsplit–plot*design using Sudoku square design structure that will capture some variability that is not accounted for by ordinary split-plot designs and also ease the randomization process.TheLinear models of the proposed design are presented with their ANOVA tables. The computational procedures along with some numerical examples were also discussed.The design is recommended when there are more than two factors and one or more of the factors are hard–to–change factors that are expensive or time-consuming to change the level settings and other factors whose level settings are easier to change. The proposed design*is illustrated with the help of a numerical example.

Key words: Sudoku, Whole-plot, Sub-plot

INTRODUCTION

Split–plot originally developed byFisher (1925) for use in agricultural experiments is basically the modified form of randomized block designs. This designinvolved one set of factors with a large number of experimental materials (whole–plot factors) and another set of factors with smaller experimental materials (subplot factors). In other words, split–plot design involves two different types of experimental units (large or small) randomly assigned independently at the two different levels (Nuga*et al.,* 2014). A Split–plotexperimental design was recently studied by Hu and Zhao (2022)who considered the situation where the selection ofthe levels of the whole-plot (WP) factors can affect that of the sub-plot (SP) factors in some experimentsand proposed a new optimality criterion for selecting suchFractional factorial split-plot(FFSP)designs. The robustness of their proposed method was discussed. The construction methodof the optimal designs under the new criterion was also been studied.The creation of ideal regular two-level fractional factorial split-plot designs was taken into consideration by Han and Zhao (2022). For two distinct design contexts, the optimality criteria were put out. Theoretical building methods for the best regular two-level fractional factorial split-plot designs were then put forth in accordance with the established optimality criteria. Theoretical construction techniques of a few ideal regular two-level fractional factorial split-plot designs were also investigated under the widely used universal minimum lower-order confounding criterion.

Sudoku square design deals with treatments that are arranged in a square array such that each row, column, or the box (sub-block) contains only one treatment. Hui-Dong and Rui-Gen (2008) observed that the Sudoku square designs go beyond Latin square designs with one additional source of variation which is the box effect. Subramani and Ponnuswamy (2009) extend the work Hui-Dong and Ru-Gen (2008) with additional terms in the model and in the sources of variation that is row-blocks and column-blocks effect Sudoku squares have been widely used to design an experiment where each treatment occurs exactly once in each row, column or sub-block. For some experiments, the size of the row (or column or sub-block) may be less than the number of treatments.

Many researchers addressedvarious classical problem of experimental designs, in the construction of ordinary designs using either Latin square or Sudoko squares designs structures. This new development frequently captures some variability that are not been accounted for by ordinary designs and also allowseach treatment of the ordinary experimental designs tooccur exactly once in each row, column or sub-block. On this note, the analysis of variance (ANOVA) method using Sudoku square models was studied by Shehu and Danbaba (2018a). The parameters, computational procedures, and the analysis of the variance table of the new design were derived.Similarly,the inclusion of concomitant variables into existing Sudoku square design models was used to obtain an analysis of the covariance of Sudoku square design models. The least square method was used for the derivation of the sum of squares and cross products for the various effects of the proposed models as well as analysis of covariance of the Sudoku square design (Shehu and Danbaba, 2018b).

Shehu and Danbaba (2018c) developed the multivariate extension of the analysis of variance (MANOVA) for the Sudoku square design and its generalized linear model. The significant tests were carried out at 0.05 alpha level of significance and the result shows that the effects are more significant than that of Hui-Dong and Ru-Gen models I and II.Mingyao*et al.,*(2013) proposed and discussed a general method for constructing BILS for the selection of certain cells from a complete Latin square via orthogonal Latin squares. The optimality for BILS designs was investigated and revealed that the transversal BILS designs are asymptotically optimal for all the row, column, and treatment effects. Danbaba*et al.,* (2018) presented a joint analysis of several experiments conducted via orthogonal Sudoku square design of odd order and concludethat joint analysis of multi-environment experiments increases the accuracy of evaluation, instead of analyzing each experiment separately.

Kaur and Garg (2020)studiedthe association among the elements of a Sudoku square, where an association scheme is defined and incomplete Sudoku square designs which are capable of studying four explanatory variables and also happen to be the designs for two-way elimination of heterogeneity are constructed. Some series of Partially Balanced Incomplete Block (PBIB) designs have also been obtained.Dauran*et al.,* (2020) consider the work of Mingyao*et al.,*(2013) and proposed a general method for constructing balanced incomplete Sudoku squares design (BISSD) by an intelligent selection of certain cells from a complete Latin square via orthogonal Sudoku designs. The relative efficiencies of a delete-one-transversal balance incomplete Latin Square (BILS) design with respect to the Sudoku design were derived and numerical examples and procedures for the analysis of data for BISSD were also discussed.Bansaland Garg (2022) consider an *m*-associate class association scheme and propose new series of Partially Balance Incomplete Block (PBIB) designs by converting a generalized $ab \times ab$ Sudoku square design with internal boxes of order $a \times b$, into a Youden-*m* square or an incomplete Sudoku square design which are competent to study three and four predictor variables respectively. The computational procedures, the linear model of the design, and some numerical examples were obtained.

However, in a situation where there existsone or more of the factors that are hard–to–change that are expensive or time-consuming to change the level setting, and other factors whose level settingsare easier to change, the works ofthe aforementioned authors cannot be applied. Therefore, it is based on this background this study came up with the construction and analysis of split–plot designs using the Sudoku square design's structure. These designs will capture some variability that are not been accounted for by ordinary split-plot designs and it will also ease the randomization process.

METHODOLOGY

Construction of Split–Plot Design with Sudoku Square Structure where Whole-plot is Completely Randomized

In the split-plot design, the whole plot treatments effectively act as blocks for the split-plot treatments so we would expect that the analysis would be largely unaffected by the design on the whole plot treatments. Consider an experimental design involving a Sudoku square design of order $m^2 (m = 2,3,4, \dots)$. Suppose that one or more of the factors are hard-to-change factors or factors that are impossible to change level setting. Then set each row-blocks or column-blocks as whole plot in the split plot design. Randomly assign the whole plot treatments levels to each row-blocks or column-blocks. The sub-blocks (sub-squares) is the replications within the row-blocks, rows or columns within the sub-squares and the treatments within the sub-squares (A, B, C, D, …) are the split-plot treatments.

One possible arrangement is given in figure 1, with 9×9 – Sudoku–square, column–blocks act as whole–plots and columns within the column–blocks act as replications within the whole plot, row-blocks, rows within the row-blocks, treatments (A, B, C, D, E, F, G, H, I) within the sub-squares are the split-plot treatments

WHOLE-PLOT		CB 1			CB 2			CB 3		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
RB1	R1	C	A	B	F	D	E	I	G	H
	R2	F	D	E	I	G	H	C	A	B
	R3	I	G	H	C	A	B	F	D	E
RB2	R1	A	B	C	D	E	F	G	H	I
	R2	D	E	F	G	H	I	A	B	C
	R3	G	H	I	A	B	C	D	E	F
RB3	R1	B	C	A	E	F	D	H	I	G
	R2	E	F	D	H	I	G	B	C	A
	R3	H	I	G	B	C	A	E	F	D

Figure 1: Split-plot arrangement with 9 × 9 – Sudoku–square

Linear Model for Split-plot Design using Sudoku Square Design Structure with Whole-plot Completely Randomized

The model in equation (1) is the linear model for split-plot design using Sudoku square design structure where whole-plot is completely randomized. The distribution assumptions, derivation of sum-of-squares as well as the ANOVA table are given below:

$$Y_{ij(klq)} = \mu + \alpha_i + \pi_{ij} + \tau_k + (\alpha\tau)_{ik} + \gamma_l + \delta_q + e_{ij(klq)} \quad (1)$$

The index $i, j, k, l = 1,2, \dots, m$ and $q = 1,2, \dots, m^2$

Where $\mu = General\ mean\ effect$

$\alpha_i = i^{th}\ column - block\ effect\ (whole - plot\ treatments)$

$\pi_{ij} = whole - plot\ level\ error, \quad \pi_{ij} \xrightarrow{iid} N(0, \sigma_\pi^2)$

$\tau_k = k^{th}\ treatment\ effect\ \ (split - plot\ treatments)$

$(\alpha\tau)_{ik} = WP\ Tr - SP\ Tr\ interaction$

$\gamma_l = l^{th}\ row\ effect$

$\delta_q = q^{th}\ Sudoku\ treatment\ effect$

$e_{ij(klq)} = \ split - plot\ error\ , \quad e_{ij(kl)} \xrightarrow{iid} N(0, \sigma_e^2)$, independent of π_{ij}

$N = m^4$

After a little algebra we have obtained the formula for various sum-of-squares with degrees of freedom and formed the analysis of variance table.

$$\mu = \frac{\mathrm{G}}{\mathrm{N}}\ , \qquad where N = m^4\ and\ G = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} \mathrm{Y_{ij}}$$

$$SST = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{ij}^2 - \frac{G^2}{N}$$

$$SSW = \frac{1}{m^3}\sum_{i=1}^{m} \alpha_i^2 - \frac{G^2}{N}$$

$$SSWE = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} Y^2{}_{ij.} - \frac{1}{m^3}\sum_{i=1}^{m} \alpha_i^2$$

$$SSTr = \frac{1}{m^3}\sum_{k=1}^{m} \tau_k^2 - \frac{G^2}{N}$$

$$SSI = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{k=1}^{m} Y_{i.k}^2 - \frac{1}{m^3}\sum_{i=1}^{m} \alpha_i^2 - \frac{1}{m^3}\sum_{k=1}^{m} \tau_k^2 + \frac{G^2}{N}$$

$$SSR = \frac{1}{m^3}\sum_{l=1}^{m} \gamma_l^2 - \frac{G^2}{N}$$

$$SStr = \frac{1}{m^2}\sum_{q=1}^{m^2} \delta_q^2 - \frac{G^2}{N}$$

$$SSE = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m^2} Y^2{}_{ijk} - \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} Y^2{}_{ij.} - \frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{m^2} Y_{i.k}^2 + \frac{1}{m^3}\sum_{i=1}^{m} \alpha_i^2 - \frac{1}{m^3}\sum_{l=1}^{m} \gamma_l^2 - \frac{1}{m^2}\sum_{l=1}^{m^2} \delta_q^2 + 2\frac{G^2}{N}$$

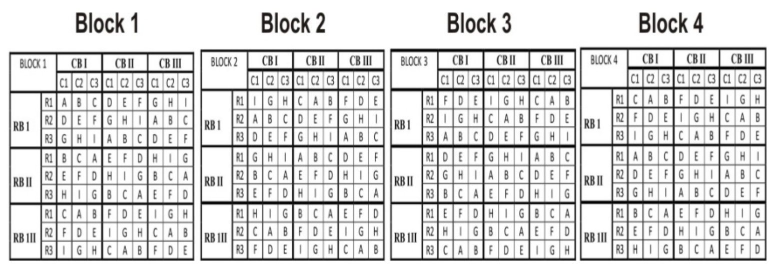
Table 1: ANOVA table for split–plot design with Sudoku structure (CRD on the WP)

Source	Sum of squares	Degrees of Freedom	Mean squares	F-Ratio (observed)
Column–Blocks (WP Treatment)	$SSCB$	$m-1$	$MSCB = {}^{SSCB}/_{df}$	${}^{MSCB}/_{MSWE}$
WP error	$SSWE$	$(m-1)(m-1)$	$MSWE = {}^{SSWE}/_{df}$	
SP Treatments (RB)	$SSRB$	$m-1$	$MSRB = {}^{SSRB}/_{df}$	${}^{MSRB}/_{MSE}$
$WP\ Tr \times SP.Tr\ interaction$	SSRBTr	$(m-1)(m-1)$	$MSRBTr = SSRBTr/df$	$MSRBTr/MSE$
Row	SSR	$m-1$	$MSR = \frac{SSR}{df}$	MSR/MSE
Treatment effect	SStr	m^2-1	$MStr = \frac{SStr}{df}$	$MStr/MSE$
Error	SSE	By subtraction	$MSE = {}^{SSE}/_{df}$	
TOTAL	SST	m^4-1		

Construction of Split–Plot Design using Sudoku Square Design Structure with Blocking on the Whole-plot

In this design, the aim is to have the whole plot in a Randomized Complete Block (RCB). By construction, the split-plot treatments are already in an RCB with the whole plots as blocks. Suppose an experimenter divide the land into n-whole-plots such that each whole-plot consists of Sudoku square design of order $m^2 (m = 2,3,4, \dots)$ and suppose that one or more of the factors are hard-to-change factors or factors that are impossible to change level setting. The whole-plot treatments are assigned at random to row-blocks or column-blocks. Randomly assign other treatments with a separate and independent randomization in each row-block or column-block. The sub-blocks (sub-squares), rows or columns within the sub-squares and the treatments within the sub-squares (A, B, C, D, …) are the split-plot treatments.

One possible arrangement is given in figure 2, with 4 blocks of 9×9 Sudoku square designs act as blocks, column–blocks is the whole plot treatment and 3×3 sub-squares, rows within the sub-squares, treatments within the sub-squares (A, B, C, D, E, F, G, H, I) are the split-plot treatments



Linear Model for Split-plot with Sudoku Square Design Structure and Blocking on the Whole-plot

The model in equation (2) is the linear model for split-plot design using Sudoku square design structure where Randomized Complete Block (RCB) will be on the whole-plot. The distribution assumptions, derivation of sum-of-squares as well as the ANOVA table are given below:

$$Y_{ij(kpq)} = \mu + \alpha_i + \beta_j + \pi_{ij} + \tau_k + (\alpha\tau)_{ik} + (\beta\tau)_{jk} + C_p + \delta_q + e_{ij(kpq)} \qquad (2)$$

The index $i,k,p = 1,2,\dots,m$, $j = 1,2,\dots,n$ and $q = 1,2,\dots,m^2$

Where $\mu = General\ mean\ effect$

$\alpha_i = i^{th}\ row - block\ effect\ (whole - plot\ treatments)$

$\beta_j = j^{th}\ blocks\ effect, \beta_j \overset{iid}{\rightarrow} N(0, \sigma_\beta^2)$

$\pi_{ij} = whole - plot\ level\ error\ (RB - CB\ interaction)$, $\pi_{ij} \overset{iid}{\rightarrow} N(0, \sigma_\pi^2)$

$\tau_k = k^{th}\ treatment\ effect\ \ (split - plot\ treatments)$

$(\alpha\tau)_{ik} = fixed\ interaction\ (at\ SP\ level)$

$(\beta\tau)_{jk} = random\ interaction\ (at\ SP\ level)$, $(\beta\tau)_{jk} \overset{iid}{\rightarrow} N(0, \sigma^2_{\beta\tau})$

$C_p = p^{th}\ Column\ effect$

$\delta_q = q^{th}\ Treatment\ effect$

$e_{ijk} = split - plot\ error$, $e_{ijk} \overset{iid}{\rightarrow} N(0, \sigma_e^2)$, independent of π_{ij}

$N = nm^4$

After a little algebra we have obtained the formula for various sum-of-squares with degrees of freedom and formed the analysis of variance table.

$$\mu = \frac{G}{N}, \quad where N = nm^4\ and\ G = \sum_{i=1}^{m^2}\sum_{j=1}^{n}\sum_{k=1}^{m^2} Y_{ijk}$$

$$SST = \sum_{i=1}^{m^2}\sum_{j=1}^{n}\sum_{k=1}^{m^2} Y^2{}_{ijk} - \frac{G^2}{N}$$

$$SSW = \frac{1}{nm^3}\sum_{i=1}^{m}\alpha_i^2 - \frac{G^2}{N}$$

$$SSB = \frac{1}{m^4}\sum_{j=1}^{n}\beta_j^2 - \frac{G^2}{N}$$

$$SSWE = \frac{1}{m^3}\sum_{i=1}^{m}\sum_{j=1}^{n} Y^2{}_{ij.} - \frac{1}{nm^3}\sum_{i=1}^{m}\alpha_i^2 - \frac{1}{m^4}\sum_{j=1}^{n}\beta_j^2 + \frac{G^2}{N}$$

$$SSTr = \frac{1}{nm^3}\sum_{k=1}^{m}\tau_k^2 - \frac{G^2}{N}$$

$$SSI_1 = \frac{1}{nm^2}\sum_{i=1}^{m}\sum_{k=1}^{m} Y_{i.k}^2 - \frac{1}{nm^3}\sum_{i=1}^{m}\alpha_i^2 - \frac{1}{nm^3}\sum_{k=1}^{m}\tau_k^2 + \frac{G^2}{N}$$

$$SSI_2 = \frac{1}{m^3}\sum_{j=1}^{n}\sum_{k=1}^{m} Y_{.jk}^2 - \frac{1}{m^4}\sum_{j=1}^{n}\beta_j^2 - \frac{1}{nm^3}\sum_{k=1}^{m}\tau_k^2 + \frac{G^2}{N}$$

$$SSC = \frac{1}{nm^3}\sum_{p=1}^{m} C_p^2 - \frac{G^2}{N}$$

$$SStr = \frac{1}{nm^2}\sum_{q=1}^{m^2}\delta_q^2 - \frac{G^2}{N}$$

$$SSE = \sum_{i=1}^{m^2}\sum_{j=1}^{n}\sum_{k=1}^{m^2} Y^2{}_{ijk} - \frac{1}{m^2}\sum_{i=1}^{m^2}\sum_{j=1}^{n} Y^2{}_{ij.} - \frac{1}{n}\sum_{i=1}^{m^2}\sum_{k=1}^{m^2} Y_{i.k}^2 - \frac{1}{m^2}\sum_{j=1}^{n}\sum_{k=1}^{m^2} Y_{.jk}^2 + \frac{1}{nm^2}\sum_{i=1}^{m^2}\alpha_i^2 + \frac{1}{m^4}\sum_{j=1}^{n}\beta_j^2 + \frac{1}{nm^2}\sum_{k=1}^{m^2}\tau_k^2 - \frac{1}{nm^3}\sum_{p=1}^{m} C_p^2 - \frac{1}{nm^2}\sum_{q=1}^{m^2}\delta_q^2 + \frac{G^2}{N}$$

Table 2: ANOVA table for split–plot design using Sudoku structure (RCB on the WP)

Source	Sum of squares	Degrees of Freedom	Mean squares	F-Ratio (observed)
Blocks	SSB	$n-1$	$MSB = SSB/df$	
Column–Blocks (WP Treatment)	$SSCB$	$m-1$	$MSCB = {SSCB}/{df}$	${MSCB}/{MSWE}$
WP error	$SSWE$	$(n-1)(m-1)$	$MSWE = {SSWE}/{df}$	
$Rows\ (SP\ Treatment)$	$SSSP$	(m^2-1)	$MSSP = SSSP/df$	$MSSP/MSE$
$Rows \times CB\ interaction$	SSRCB	$(m^2-1)(m-1)$	$MSRCB = {SSRCB}/{df}$	${MSRCB}/{MSE}$
$Blocks \times Rows$	SSBR	$(n-1)(m^2-1)$	$MSBR = {SSBR}/{df}$	${MSBR}/{MSE}$
Column effect	SSC	$m-1$	$MSC = \frac{SSC}{df}$	MSC/MSE
Treatment effect	SStr	m^2-1	$MStr = \frac{SStr}{df}$	$MStr/MSE$
Error	SSE	By subtraction	$MSE = {SSE}/{df}$	
TOTAL	SST	nm^4-1		

Analysis of Split-Plot with Sudoku Square Design Structure Where Whole-plot is Completely Randomized

In this section, the analysis of split-plot design with Sudoku square structure is explained with the help of numerical example. For this purpose, hypothetical datasets were generated using random number table and are given in figure 3 below.

In this analysis, column blocks act as the whole-plot treatments, columns as replication within the whole plots and row-blocks as split-plot treatments. The result is displayed in table 3.

		CB I			CB II			CB III		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
RB 1	R1	22	17	23	20	21	17	25	18	20
	R2	19	19	21	25	22	20	23	19	23
	R3	23	22	25	22	24	22	17	20	23
RB II	R1	18	22	24	17	17	18	22	19	24
	R2	20	25	23	22	18	17	24	19	17
	R3	20	19	17	23	23	24	18	17	23
RB 1II	R1	19	25	20	25	21	17	24	20	24
	R2	19	19	17	25	23	25	24	17	23
	R3	23	19	19	21	23	24	20	18	23

Figure 3: Hypothetical data of a Sudoku square design of order 9

Table 3: ANOVA table for the hypothetical data of Split-plot design using Sudoku square designs structure (CRD on the whole plot)

Source	DF	SS	M.S	F-ratio	P_value
WP Treatment	2	5.6543	2.8272	0.1879	0.8334
WP Error	6	90.2963	15.0494	2.5536	0.0295
SP Treatment	2	15.2840	7.642	1.2967	0.2815
WP_SP Interaction	4	32.3457	8.0864	1.3721	0.2553
Row effect	2	30.1728	15.0864	2.5599	0.0863
Treatment effect	8	68.1728	8.5216	1.446	0.1982
SP Error	56	330.0247	5.8933	-	-
Total	80	571.9506	-	-	-

Analysis of Split-Plot Design with Sudoku Square Structure and Blocking on the Whole-plot

The analysis of split-plot design with Sudoku square structure and Randomized Complete Block (RCB) on the whole-plot is explained with the help of numerical example. For this purpose, the hypothetical data were generated using random number table and are given in figure 4 below.

Block 1

		CB I			CB II			CB III		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
RB 1	R1	22	17	23	20	21	17	25	18	20
	R2	19	19	21	25	22	20	23	19	23
	R3	23	22	25	22	24	22	17	20	23
RB II	R1	18	22	24	17	17	18	22	19	24
	R2	20	25	23	22	18	17	24	19	17
	R3	20	19	17	23	23	24	18	17	23
RB 1II	R1	19	25	20	25	21	17	24	20	24
	R2	19	19	17	25	23	25	24	17	23
	R3	23	19	19	21	23	24	20	18	23

Block 2

		CB 1			CB II			CB III		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
RB I	R1	18	25	18	18	25	20	24	24	24
	R2	19	21	18	17	19	19	22	23	25
	R3	19	20	24	17	23	17	25	17	19
RB II	R1	21	25	21	20	25	17	25	25	25
	R2	21	20	19	20	24	19	25	19	19
	R3	22	22	22	23	24	20	21	25	25
RB III	R1	20	21	17	17	17	21	24	20	24
	R2	21	24	25	24	24	18	23	19	23
	R3	23	19	18	25	17	22	25	18	19

Block 3

		CB I			CB II			CB III		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
RB 1	R1	21	24	19	19	19	18	18	22	23
	R2	25	21	20	23	25	18	22	19	18
	R3	19	23	22	21	19	17	23	24	18
RB II	R1	24	22	24	25	21	25	19	25	24
	R2	17	20	25	25	25	20	20	23	25
	R3	22	21	24	18	22	19	19	17	17
RB 1II	R1	22	18	17	19	20	22	24	19	25
	R2	25	25	18	21	25	24	19	23	18
	R3	19	18	19	19	17	24	19	17	20

Block 4

		CB I			CB II			CB III		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
RB 1	R1	17	25	24	23	25	24	25	20	21
	R2	25	18	25	21	25	24	21	20	20
	R3	22	21	17	25	20	17	25	22	19
RB II	R1	24	18	21	23	22	17	21	18	19
	R2	19	23	23	17	22	24	21	25	21
	R3	17	17	23	22	17	22	21	24	21
RB 1II	R1	24	23	21	25	24	23	23	19	20
	R2	21	19	17	17	17	18	22	19	21
	R3	22	18	21	24	23	18	22	22	18

Figure 4: Hypothetical data for Sudoku square design with 4 blocks (RCB on the Whole-Plot)

For this analysis, the blocks in figure 4 acts as blocks, column-blocks within the blocks acts as whole-plot treatments and rows within the row-blocks act as split plot treatments. The result is displayed in table 4.

Table 4: ANOVA table for the result of Split-plot design using Sudoku square design structure (RCB on the whole-plot)

Source	DF	SS	M.S	F-ratio	P_value
Blocks	3	3.9105	1.3035	2.1751	0.2404
WP Treatment	2	7.5062	3.7531	6.2627	0.0674
WP Error	6	3.5957	0.5993	0.0796	0.9981
SP Treatment	2	30.6358	15.3179	2.0337	0.1311
WP_SP Interaction	4	23.1296	5.7824	0.7677	0.5462
SP_Bl Interaction	6	26.7253	4.4542	0.5914	0.7375
Row-Block Effect	2	4.8951	2.4475	0.325	0.7226
Column Effect	2	93.0247	46.5123	6.1754	0.0021
Treatment Effect	8	77.6358	9.7045	1.2885	0.2448
SP Error	288	2169.1852	7.5319	-	-
Total	323	2347.2191	-	-	-

A. ***Conclusion***

In this study, a newconstruction and analysis of split-plot designs using Sudoku square design structure has been extensively studied under two different randomization scenarios on the whole-plot (CRD and RCB). These designs can be used when there are more than one factors and one or more of the factors are hard-to-change factors that are expensive or time consuming to change level setting and the other factors whose level setting are easier to change.These designs had captured some variability that is not been accounted for by ordinary split-plot designs. Also, from the results of the hypothetical data it has been shown that these designs have minimum mean square error compared to the ordinary split-plot design. Therefore, it is more efficient than the ordinary split-plot design.

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