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Dark energy cosmological model with anisotropic fluid and time-varying lambda in Kaluza-Klein metric

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***Abstract:*In this paper, we have investigated the dark energy cosmological model in presence of anisotropic fluid in Kaluza–Klein metric with generalized time-dependent lambda $\Lambda=\alpha H^2 + \beta R^2$ (α, β are free parameters; H is Hubble parameter). Considering the equation of state (EOS) $p = \omega\rho$ for normal dimensions and $p_\psi=(\omega+\delta)\rho$ for the fifth dimension, exact solutions of Einstein field equations of the anisotropic model are obtained (where p - the pressure for normal dimensions, p_ψ – the pressure of the fifth dimension, ρ - density of the fluid, ω - EOS parameter, and δ - skewness parameter). It is concluded that the universe at its early stage shows anisotropic behavior due to its finite value δ. The variations of ω and δ demonstrate the evolution from radiation dominated early universe to a dark energy-dominated universe. We have also investigated dark energy density, pressure, and other physical parameters. The physical parameters are dependent on free parameters and power index factor n which relates the extra dimension scale factor to the normal scale factor.**

***Index Terms:*Dark energy; Cosmological constant; Equation of state; Kaluza-Klein Cosmological model;Exact solution.**

I. INTRODUCTION

The present arena of the universe showcases that it consists of 68% of dark energy(DE), 28% dark matter(DM), and 4% visible matter. The existence of dark energy has been confirmed through the accelerated expansion of the universe which has been illustrated by the High-Z supernova search team led by Reiss et al (2004) and the supernova cosmology project headed by Perlmutter et al (1997). Recently, the expected data through Dark Energy Spectroscopic Instrument (DESI) survey explains the nature of dark energy (Jaffe et al, 2001; Pryke et al, 2002; Seo &Eisenstein, 2003) which is yet to be cracked. Another constitute of the universe, dark matter (DM) is also a mystery. Zwicky (1937) has identified the discrepancy between observed and predicted galactic rotational curves and suggested that it could be due to the presence of DM. In the present scenario, the existence of DE and DM has been well established, but the mystery of their nature remains unsolved. This has prompted many cosmologists to explore the study of models with dark energy fluid. Cheng et al (2020), and Jain (2020) have studied dark energy–dark matter interactive models with varied perspectives. These interactive DE-DM models are studied extensively nowadays.

This paper explores a cosmological model with a generalized time-dependent cosmological constant (Λ) in the presence of anisotropic dark energy fluid in the Kaluza-Klein metric.

Cosmological studies have been revealed by assuming the universe consists of fluid modeled with the equation of state (EOS) $\omega = p/\rho$ (p - pressure, ρ - density of the fluid), the values of which have been utilized to study different phases of the universe (Alcaniz, 2006). In this regard, DE EOS ω is considered to be equal to -1. Jimenez,2003; Usmani, et al,2003;& Amendola,(2003)have suggested quintessence and phantom forms of dark energies models with $\omega > -1$ & $\omega < -1$ respectively. Gorbunova & Timoshkin (2008), and Das et al,(2005)have proposed theoretical models with time-dependent ω which is yet to be confirmed experimentally. Melia (2015) put forward supernova cosmological project results with the new constraints on ω given by $\approx -1.05 \pm .09$ for a flat universe. The experimental observations for luminosity distance, high redshift, and galaxy clustering conclude with values of ω as $-1.44 < \omega < -0.92$ at 68% confidence level (Knop et al, 2003; Hinshaw et al, 2013). In this regard, Wellar & Lewis (2003) have concluded with similar results. These constraints on depicting the existence of dark energy and the variation of pressure in different directions bring out the anisotropic nature of the universe. The presence of a small anisotropy in the current isotropic scenario has also been concluded through the study of minute temperature variations observed at the Cosmic Microwave Background (CMB) level (Spergel et al,(2007). This is supposed to contain information about early universe phenomena, phase transition, etc. Microwave Anisotropy Probe (MAP) (Hinshaw et al, 2009) and COBRAS-SAMBA (Planck surveyor)(Bouchet et al,1995), BAO-SDSS (Baryon Acoustic Oscillation –Sloan Digital Sky Survey) (Eisenstein et al,2005)experimental results led to a further study of anisotropy in CMBR radiations.

Various Bianchi-type anisotropic dark energy models, FRW isotropic & anisotropic models have been dealt with by studying the nature of dark energy in the current scenario (Aakarshu &Kilinc,2010; Middleton & Stanley, 2011; Pradhan, et al 2011; Saha,2014). Yadav et al, 2011; Mahanta 2013, Mukhopadhyay, et al,2007;& Johri & Rath, 2007 have explained the dynamics of dark energy in the anisotropic universe, considering parameterized ω (t) introducing skewness parameters (δ) in different directions.

Berman, 1983; Koivisto & Mota, 2006 investigated the models with variations of the Hubble parameter in absence of a cosmological constant (Λ). Several other forms of dark energy models in absence of cosmological constant have also been investigated in quintessence, braneworld, f(R) gravity, scalar field, etc. (Adhav et al, 2011; Mishra et al,2017; Sahni & Starobinsky,2006; Ratra & Peebles,1988)

Recent studies and experimental observations in cosmology do consider the significance of the cosmological constant that has been first introduced by Einstein and is now known to be physically significant with dark energy (Carroll et al, 1992). An excellent review of the cosmological constant by Weinberg,(1989) and Sahni,(2002) has revealed that the cosmological constant has been plagued with a cosmological constant puzzle (CCP). The CCP has a discrepancy of about 120 orders between its cosmological observed value and the calculated one at the Planck level. Overduin & Cooperstock,1998; Chen & Wu 1990; and Carvalho et al,1992have tackled this problem by investigating 4D FRW models with time-dependent lambda varying with H^2, R^{-m}, qH^2 (q – deceleration parameter), or $\alpha H^2+\beta R^{-2}$ (generalized lambda). Here, the cosmological constant varies with the time-dependent scale factor. Considering time-dependent lambda in Einstein-Hilbert action, the diffeomorphism invariance implies the modification of Einstein field equations. But these modifications led to the explanation of the accelerated expansion of the Universe (Oztas A. M, Dil E. and Smith M., 2018 and references therein).

The dark energy model with a generalized lambda has successfully dealt with age, low-density problems, presence of anisotropy in the cosmic background of the universe. However, shortfalls of 4D models in dealing with CCP and cosmological coincidence problems have led cosmologists to search for an alternative in the higher dimensional field.

With the upsurge of string theory, 5D models have gained popularity for their simplicity which can not only explain early universe phenomena but also can depict present universe scenarios. In this regard, Kaluza,(1921) first put forth five-dimensional models to unify gravitational and electromagnetic forces, and laterKlein (1926) employed gauge theory to explain the five-dimensional theory. In this regard, Wesson P.(1999) proposed the space-time-matter theory or induced matter theory which helps in the explanation for the unification of gravity and weak forces.

The original Kaluza-Klein theory placed two very strong constraints on the fifth dimension, namely, (i) that all partial derivatives concerning the fifth coordinate are zero (cylinder condition), and (ii) that the fifth dimension has a closed short-scale topology (compactification condition). The most important consequences of these conditions are that no change in 4-dimensional physical quantities can be ascribed to the presence of an extra spatial dimension and that such a fifth dimension is unobservable at low energies. Condition (ii) was also a vital ingredient in the attempt to explain the quantization of electric charge. It is interesting to note that condition (ii) prevents whatever microscopic object from spanning the fifth dimension (Vassallo A., 2015). The extra dimensions were thought to be lower thanthe Planck scale and so could not be tested experimentally but its effect can be experienced (Tanashahi. M, 2018).

In this regard, the existence of the Kaluza-Klein particles can confirm the presence of an extra dimension. Considering the fifth dimension as a scalar function ϕ in the form of a circle of radius r and its Fourier expansion results in higher orders of the functions or towers of massive modes. These are termed Kaluza-Klein excitations or also identified as K-K particles (Melb´eus H.,2012, Bringmann T., 2005). One of the constituents of dark matter has been supposed to be K-K particles since its constituents are still unknown to us.

A rich literature on Kaluza-Klein cosmological models is now available which has been studied in various contexts. The Kaluza-Klein models with different forms of matter in presence of generalized lambda ($\Lambda = \alpha H^2+\beta R^{-2}$) have been investigated by Chodos & Detweiler,1980; Singh et al,2006; and Jain et al,2015; 2013; 2014; 2013. These models have demonstrated the effects of extra dimensions on the nature of dark energy and various physical parameters. Milton 2003& Radar 2011 have concluded the correlation between dark energy and extra dimension. Some cosmologists (Demaret & Hanquin,1985;Katore et al,2014;Adhav et al,2011) have studied Kaluza-Klein cosmology with anisotropic dark energy in absence of lambda, explaining the universe at late times. These models have been inspirational for the present study of the anisotropic model.

With the above motivation, we have investigated the cosmological model in the Kaluza-Klein metric in presence of time-varying lambda. We have examined DE cosmological model with anisotropic fluid by introducing skewness parameter δ in EOS of extra-dimension. This led to the study of directional dark energy fluid. This paper is organized into six sections. With the introduction in the first section; metric and field equations are discussed in section 2. Solutions of field equations and some physical parameters are obtained in sections 3 and 4 respectively, followed by discussion and conclusion in sections 5 and 6 respectively.

II. Metric and Field equations

To find the Einstein field equation, we consider the Kaluza-Klein metric (Jain et al,2013) as given below

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] + A^2(t)d\psi^2. \quad (1)$$

Here $R(t)$, and $A(t)$ is fourth and fifth-dimension scale factors respectively, k is the curvature constant which is equal to 0,1 and -1 for flat, closed, and open universes respectively., and energy-momentum tensor for the anisotropic model (Adhav et al,2011)] for the above metric is given as below,

$$T_j^i = diag(T_0^0, T_1^1, T_2^2, T_3^3, T_4^4) = diag\,(-\rho, p, p, p, p_\psi)(2)$$

Whereρ, p and p_ψ are the density, pressure, and extra dimension pressure of the DE fluid respectively. EOS for normal dimensions has been assumed to be p = ωρ,ω is the equation of state parameter (for normal dimensions), while p_ψ = (ω+δ)ρ for extra-dimension, and is the skewness parameter introducing the deviation from isotropy. Hence,

$$T_j^i = diag\,(-\rho\,, \omega\rho, \omega\rho, \omega\rho, (\omega+\delta)\rho) \quad (3)$$

ω and δ may not necessarily be constants.

Einstein's field equations are arrived at by the following equation:

$$G_j^i = R_j^i - \frac{1}{2}Rg_j^i = -8\pi G T_j^i + \Lambda g_j^i\,. \quad (4)$$

In deriving Einstein's field equation we assume $8\pi G = c = 1$ and using *ansatz* for metric potentials [Jain et al, 2014 and references therein] $A = R^n$, field equations derived from Eq.(4) are given below :

$$G_1^1 = (n+2)\frac{\ddot{R}}{R} + (n^2+n+1)\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -p + \Lambda, (5)$$

$$G_4^4 = 3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = -p_\psi + \Lambda \quad (6)$$

$$G_5^5 = 3(n+1)\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = \rho + \Lambda \quad (7)$$

These field equations are rewritten as,

$$(n+2)\frac{\ddot{R}}{R} + (n^2+n+1)\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -\omega\rho + \Lambda\,, \quad (8)$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = -(\omega+\delta)\rho + \Lambda \quad (9)$$

$$3(n+1)\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = \rho + \Lambda\,. \quad (10)$$

Divergence of Einstein's tensor has been given by,

$$\left(R_j^i - \frac{1}{2}Rg_j^i\right)_{;j} = \left(-T_j^i + \Lambda g_j^i\right)_{;j} = 0 \quad (11)$$

From the above equation, the energy conservation equation (Adhav et al,2011) is obtained as:

$$\dot{\rho} + (\rho+p)3\frac{\dot{R}}{R} + (\rho+p_\psi)n\frac{\dot{R}}{R} + \dot{\Lambda} = 0(12)$$

Substituting p = ω ρ and p = (ω+δ) ρ in equation (11), it is further simplified as:

$$\dot{\rho} + (1+\omega)(3+n)\frac{\dot{R}}{R} + n\delta\frac{\dot{R}}{R} + \dot{\Lambda} = 0\,. \quad (13)$$

Eq. (13) can be separated into two equations of which one equation contains a deviation-free parameter and the other one has a skewness parameter so that the presence of anisotropic conditions in the present isotropic universe can be explained. The two equations are:

$$\dot{\rho} + (1+\omega)(3+n)\frac{\dot{R}}{R} = 0 \quad (14)$$

$$n\delta\frac{\dot{R}}{R} + \dot{\Lambda} = 0. \quad (15)$$

The solution of field equations is obtained in the next section by substituting $\Lambda = \alpha\frac{\dot{R}^2}{R^2} + \beta\frac{1}{R^2}$in the above equation.

III. Solution of field equations

There are three independent equations (field equations) and R, ρ, ω, δ, and Λ is five independent variables. So the solution of field equations is obtained with the help of time-dependent lambda i.e.$\Lambda = \alpha\frac{\dot{R}^2}{R^2} + \beta\frac{1}{R^2}$, where α and β are free parameters.

Subtracting equation (8) from equation (9) we get,

$$-\delta\rho = (1-n)\frac{\ddot{R}}{R} + (2-n-n^2)\frac{\dot{R}^2}{R^2} + \frac{2k}{R^2}. \quad (16)$$

Equation (15) is rewritten as,

$$n\delta\rho\frac{\dot{R}}{R} = -\dot{\Lambda}\quad . \quad (17)$$

$$\Lambda = \alpha\frac{\dot{R}^2}{R^2} + \beta\frac{1}{R^2}\,,$$

$$\dot{\Lambda} = 2\alpha \frac{\dot{R}\ddot{R}}{R^2} - 2\alpha \frac{\dot{R}^3}{R^3} - 2\beta \frac{\dot{R}}{R^3},$$

Eq. (17) is rewritten as,

$$n\delta\rho \frac{\dot{R}}{R} = -2\alpha \frac{\dot{R}\ddot{R}}{R^2} + 2\alpha \frac{\dot{R}^3}{R^3} + 2\beta \frac{\dot{R}}{R^3}. \qquad (18)$$

Rewriting the above equation as:

$$\delta\rho = -\frac{2\alpha}{n}\frac{\ddot{R}}{R} + \frac{2\alpha}{n}\frac{\dot{R}^2}{R^2} + \frac{2\beta}{n}\frac{1}{R^2}. \qquad (19)$$

From Eq. (16) and (19) we get:

$$\left[(1-n) - \frac{2\alpha}{n}\right]\frac{\ddot{R}}{R} + \left[(2-n-n^2) + \frac{2\alpha}{n}\right]\frac{\dot{R}^2}{R^2} + 2\left(\frac{\beta}{n} + k\right)\frac{1}{R^2} = 0 \qquad (20)$$

Simplifying the above Eq., we get:

$$\frac{\ddot{R}}{R} + \frac{\left[(n^2-n-2) - \frac{2\alpha}{n}\right]}{\left[(n-1) + \frac{2\alpha}{n}\right]}\frac{\dot{R}^2}{R^2} - \frac{2\left(\frac{\beta}{n}+k\right)}{\left[(n-1)+\frac{2\alpha}{n}\right]}\frac{1}{R^2} = 0. \qquad (21)$$

Assuming $m = \frac{\left[(n^2-n-2)-\frac{2\alpha}{n}\right]}{\left[(n-1)+\frac{2\alpha}{n}\right]}$, $k_1 = \frac{2\left(\frac{\beta}{n}+k\right)}{\left[(n-1)+\frac{2\alpha}{n}\right]}$, Eq.(20) is simplified as,

$$\frac{\ddot{R}}{R} + m\frac{\dot{R}^2}{R^2} - k_1\frac{1}{R^2} = 0. \qquad (22)$$

The above equation is a homogeneous second-order equation. The first-order integral equation is obtained by integrating the above equation and is given by,

$$\dot{R}^2 = A_1 R^{-2m} + \frac{k_1}{m}. \qquad (23)$$

where A_1 is the constant of integration. We consider m = -1/2 to deal with present observational data. The solution of the above equation is obtained as:

$$R(t) = \frac{2k_1}{A_1} + \frac{A_1}{4}(t+c)^2. \qquad (24)$$

In the above equation, c is the constant of integration. Constant A_1 and c canbe determined by initial conditions. At t =0 let R(t) =0 then $c = -\frac{8k_1}{A_1^2}$, Let us assume c = $-t_0$, for simplicity, Thus the above equation is rewritten as:

$$R(t) = \frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2. \qquad (25)$$

Other physical parameters are obtained in the following section.

IV. Determination of Physical Parameters

Other physical parameters i.e. H, q, ω and δ are determined using Eq. (25) as follows:

$$A(t) = R^n(t) = \left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]^n. \qquad (26)$$

$$H = \frac{A_1(t-t_0)}{2\left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]} \qquad (27)$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\left(\frac{1}{2} + \frac{4k_1}{A_1^2(t-t_0)^2}\right) \qquad (28)$$

Substituting Λ in equation (10) and rewriting it we get,

$$\rho = [3(n+1) - \alpha]\frac{\dot{R}^2}{R^2} + \frac{(3k-\beta)}{R^2} \qquad (29)$$

Using Eq. (25) in Eq.(28), we obtain,

$$\rho(t) = [3(n+1) - \alpha]\frac{A_1^2(t-t_0)^2}{4\left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]^2} + (3k-\beta)\frac{1}{\left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]^2}. \qquad (30)$$

$$\Lambda(t) = \alpha\frac{A_1^2(t-t_0)^2}{4\left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]^2} + \beta\frac{1}{\left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]^2}$$

Substituting Eq. (8) and simplifying by using Eq.(29) in it, ω is calculated as,

$$\omega = -\frac{\left[(2n^2+3n+4) - 2\alpha\right]A_1^2(t-t_0)^2 + 8k_1(n+2) + 8(k-\beta)}{2\left[(3(n+1)-\alpha)A_1^2(t-t_0)^2 + 4(3k-\beta)\right]}. \qquad (31)$$

δ is calculated from equation (16) which is given by,

$$\delta = \frac{\left[(2n^2+3n-5)A_1^2(t-t_0)^2 + 4k_1(n-1) - 8k\right]}{2\left[(3(n+1)-\alpha)A_1^2(t-t_0)^2 + 4(3k-\beta)\right]} \qquad (32)$$

Expansion factor θ and Shear scalar σ^2 are determined as given below:

$$\theta = 3\frac{\dot{R}}{R} + \frac{\dot{A}}{A} = (n+3)\,H \text{ and,}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{R}}{R} - \frac{\dot{A}}{A}\right)^2 = \frac{3}{8}(1-n)^2 H^2$$

Hence,

$$\frac{\sigma^2}{\theta} = \frac{3(1-n)^2}{8(n+3)}H = \frac{3(1-n)^2}{8(n+3)}\left[\frac{A_1(t-t_0)}{2\left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]}\right] \qquad (33)$$

V. Discussion

The present model is investigated by assuming directional EOS for normal and extra dimensions as p = $\omega\rho$ and $p_\psi = (\omega+\delta)\rho$ respectively. The solution of the Einstein field equation is obtained by assuming m = -1/2 in equation (23). From equation

(25) it is observed that $R\to\infty$ if $t\to\infty$, leads to continuous expansion.

Considering m = -1/2, it is found that $\alpha = [n\ (2n+3)\ (1-n)]/2$. Similarlyβ is calculated from k_1. For k_1 =1/2, $\beta = n(n+1)(1-n)/2 - nk$. Here there are constraints on n i.e. $n\neq$ (0, -1, 1) to have a positive value ofΛ. We have found α = 1/2 and β = -3/16 for n=-1/2. From equation (26), it is observed that extra dimension decreases rapidly with time.This results in the compactification of extra dimensions at present times.For $A_1^2/k_1 = 8$, equation (27) and equation (28) reveal that $H(t) \propto 1/t$ and $q(t) \to -1$, depict the acceleration of expanding universe which decreases with time (Fig.1 and Fig.2). From equation (30) it is observed that the density of the universe is proportional to $1/t^2$ and decreases with the advance of the time. For n=-1/2, and $t\to t_0$, $\omega \to -1$, δ has a small negative value, pointing out the presence of anisotropic dark energy fluid at present time. The following graphs are plotted for n = -1/2, A_1 = 2, and k_1 = ½ for $(t-t_0) > 0$ for the flat universe to reconcile with present observational data (Bean & Melchiorri 2002).

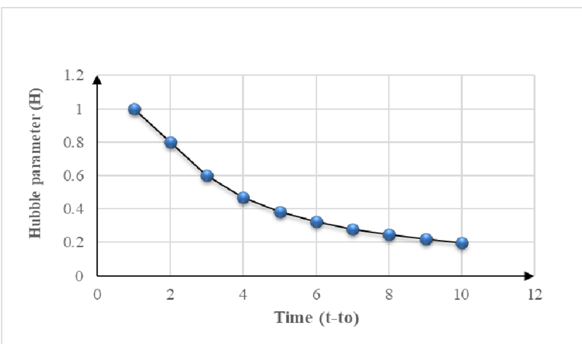

Figure 1. The plot of H v/s $(t-t_0)$

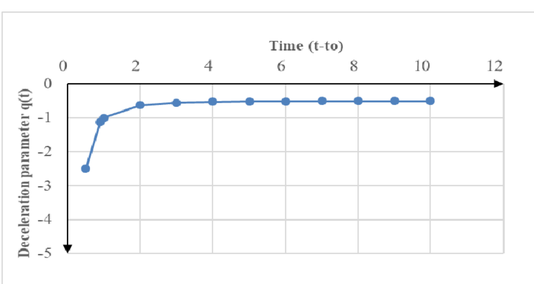

Fig. 2 Plot of q v/s $(t-t_0)$

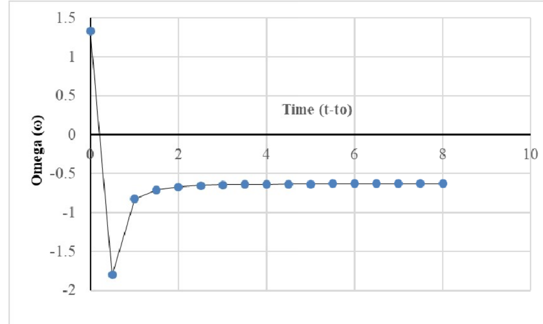

Fig. 3 Plot of ω v/s $(t-t_0)$.

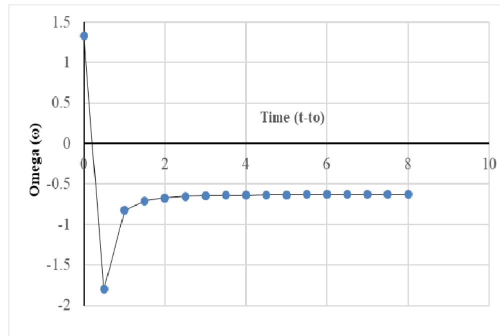

Fig.4 Plot of δ v/s $(t-t_0)$

From Fig.3 and Fig. 4, we observe that there is zero-crossing of ω and δ. This indicates the transition from radiation dominated phase to a dark energy-dominated universe. The finite but small value of δ in Fig.4 points towards the presence of slight anisotropy in the present universe. It is also observed from Eq.(33) that the anisotropy factor $\sigma^2/\theta \to 0$ when $t \to t_0$, leads to the isotropic present universe.

VI. Conclusion

Kaluza - Klein anisotropic dark energy model with time-dependent lambda has been investigated in this paper. It is found that the universe is expanding and accelerating but acceleration decreases with time. A small positive value of the cosmological constant is predicted in the present model. The joint effect of cosmological constant and deviation parameter leads to an anisotropic early universe which later evolves as the isotropic universe. It is also observed that the present universe is dominated by dark energy. The investigation of our model of the universe also demonstrates the presence of anisotropy in the present era due to the finite valueof δ. The model also explains the evolution of the universe from radiation dominated phase to a dark energy-dominated phase. Physical parameters are found to be dependent on n, free parameters β, demonstrating theimpact of extra dimension and lambda on them at present times.

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