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An Orthogonal Taylor wavelet Galerkin Numerical Method for one dimensional Partial Differential Equations

Ankit Kumar[1], Sag Ram Verma[*2]

[1]Department of Mathematics, Faculty of School of Allied Sciences, Mandsaur University, Mandsaur (M.P)-458001, India, Email: ankitkr922@gmail.com

[*2]Department of Mathematics and Statistics, Faculty of Science, Gurukula Kangri (Deemed to be University), Haridwar-249404, India, Email: drsrverma@gkv.ac.in

*Corresponding author

Abstract: **In this study, an orthogonal Taylor wavelet Galerkin numerical method with its residual process is introduced for solving one- dimensional partial differential equations which performs crucial role in electrical circuit modeling. Together, its algorithm is discussed. The goal of the introduced wavelet numerical method is to provide a fast and efficient implementation for solving one-dimensional partial differential equation. Experimentally, the introduced method is analyzed on some one-dimensional partial differential equations, and the obtaining results are compared with experimental results of other most well-known numerical methods, such as wavelets based Galerkin method, and spectral procedures, finite difference method, which indicating that the introduced method is more effective.**

Keywords: **Finite difference method; Orthogonal Taylor wavelet basis; One- dimensional partial differential equations; weight function; wavelets Galerkin methods.**

1. INTRODUCTION

Wavelet-based method has been used for better approximate numerical solution of tangled partial differential equation (PDEs), which was introduced around 30 years ago by a few mathematical researchers, and it has been extensively applied in distinct studies in the domains of mathematics [1, 2], computer science [3], mathematical physics [4, 5], and engineering [6], etc. Wavelets are a set of orthogonal functions, which are considered efficient for solving PDEs and provide best approximate solutions. Wavelet analysis contributes to a fast wavelet transform; it was introduced by Mallat [7], and compactly supported orthogonal wavelets were produced by Daubechies [7, 8]. And a wavelets approximation scheme for PDEs received momentum in pleasant way. The wavelet subject has received significant interest due to the extensive mathematical ability and good application capability of wavelets in several numerical problems. This significant interest can be attributed to the construction of the orthogonal basis of compactly supported wavelets. In recent years, different types of approximation schemes have been used to analyze the numerical results of one- dimensional PDEs; for example, finite difference methods (FDM) [9], spectral procedures [10], and finite elements methods[11]. Despite these, we need wavelet Galerkin methods (WGMs) because these have a high potential for producing more accurate results. Wavelet Galerkin methods [12-14] have enabled vast applications in computer science, applied mathematics, and engineering, and these have become more popular in the development of numerical techniques for the study of one- dimensional PDEs compared to spectral procedures or finite elements methods [9-11]. Galerkin method is an extremely good method involving the 'weighted' residual [15] for providing analytical solutions to one- dimensional PDEs. The Taylor wavelet Galerkin method [16] has emerged as an exact and efficient means of approximate solution for one-dimensional PDEs. Recently, many studies, as evidenced from the literature, have been applied wavelet- based approaches for solving boundary values problems [17, 18]. In the survey, we found that most of the physical models exist in PDEs form, and

[*] Corresponding Author

we can't always extract exact solutions for these PDEs through numerical methods. So, it is very important to present the best solution for these kinds of PDEs via wavelet- based numerical method. Furthermore, we can see that the basis function used in the finite element method (FEM) has a less compact support, and extremely weak continuity property, whereas spectral bases have global support, but they are infinitely differentiable. Similarly, spectral methods perform well in terms of spectral localization but perform poorly in terms of spatial localization, whereas FEM performs well in terms of spatial localization. And wavelet basis fulfill the specific advantages of both spectral and FEM basis. In addition, one- dimensional PDEs also occur frequently in electronics and electrical engineering [19-22]. Particularly, resistor (or capacitor) networks model is defined by the well-known Laplace equation in the field of electronic and electrical circuits, which is a well-known second-order PDE. Resistor (or Capacitors) network plays an important role in basic electronics. Figure.1, illustrates the Resistor/Capacitors networks.

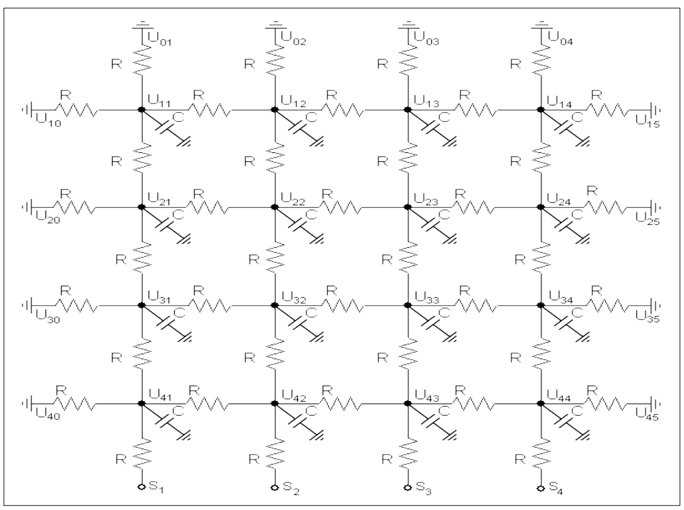

Figure 1: Resistor/Capacitor network [19].

In this work, the main objective is to give better approximate solution for one- dimensional PDEs. An efficient orthogonal Taylor wavelet Galerkin numerical method is introduced for solving this kind of one- dimensional PDEs. In this method orthogonal Taylor wavelets (OTWs) basis is new, which is to combine Galerkin method in solving one- dimensional PDES. This method is directly based on series approximation for the numerical study by OTWs with unknown parameters. Galerkin method is extremely good method of the 'weighted' residual for calculating numerical solution to one- dimensional PDEs. The proposed method accuracy can be found clearly for solving one-dimensional PDEs, which is demonstrated in numerical test problems.

Rest part of our paper arranged in following way, Section 2 deal with basic definition of wavelets and OTWs. Function approximation is given in section 3 and Section 4 consist procedure of solution of orthogonal Taylor wavelet Galerkin numerical method (OTWGNM). In section 5, Numerical problems are demonstrates. Finally, a significance and conclusion of the propose work is discussed.

2. Basic Definition of Wavelets

A family of functions, which is got from the single function by dilation (scaling) and translation, is named mother wavelet (or wavelet). If the dilation (scaling) parameter s and translation (sifting) parameter r vary continuously then we get the family of continuous wavelets [23] as

$$\psi_{s,r}(z) = |s|^{-1/2}\psi\left(\frac{z-r}{s}\right); \qquad s, r \in R;\ s \neq 0, \tag{1}$$

and, if both scaling and sifting parameters s and r replace by discrete values as $s = s_0^{-d}$, $r = m\, r_0\, s_0^{-d}$, $s_0 > 1, r_0 > 0$ and $m, d \in Z^{+}$ then it is recognized as discrete wavelet. The family of discrete wavelets is written with the help of above equation as

$$\psi_{d,m}(z) = \left|s_0^{-d}\right|^{-1/2}\psi\left(\frac{z - m\, r_0\, s_0^{-d}}{s_0^{-d}}\right) = |s_0|^{d/2}\psi\left(s_0^{d} z - m\, r_0\right), \tag{2}$$

where, $\psi_{d,m}(z)$ is wavelet basis in $L^2(R)$.

Remark: Since Taylor wavelet [23] is not orthogonal in interval $[0,1]$, so we orthogonalized it by using the orthogonalization processes of Gram-smith on normal Taylor polynomial $\widetilde{T}_q(z) = \sqrt{2q+1}\, T_q(z)$ where, $\sqrt{2q+1}$ normality coefficient and $T_q(z) = z^q$ is Taylor polynomial of order q .See ref. [24].

2.1 ORTHOGONAL TAYLOR WAVELETS

Orthogonal Taylor wavelets (OTWs) $\psi_{p,q}(z)=\psi(d, \hat{p}, q, z)$ have four arguments same as Taylor wavelets: $\hat{p} = p-1, p = 1, 2, ..., 2^{d-1}, d \in Z^{+}$, where $q = 0, 1, ..., Q-1$ is the order and z is normalized time for orthogonal Taylor polynomials (OTPs) which is defined on interval $[0,1]$ as

$$\psi_{p,q}(z) = \begin{cases} \sqrt{2^{d-1}}\; U_q(2^{d-1}z - p + 1), & \text{if } \dfrac{p-1}{2^{d-1}} \le z < \dfrac{p}{2^{d-1}}, \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $U_q(z)$ are OTPs defined on $[0,1]$ can be calculated by using the orthogonalization processes of Gram-smith on $\widetilde{T}_q(z)$ or directly obtained by the following relation:

$$U_q\left(2^{d-1}z - p + 1\right) = (2q+1)^{\frac{1}{2}}\left(\frac{(q!)^2}{(2q)!}\right)L_q\left(2^{d}z - 2p + 1\right), \tag{4}$$

where, symbol ! represent factorial sign and $L_q(z)$ is well known Legendre polynomial [25].

$$\psi_{1,0}(z)=L_0(2z-1)=1,$$
$$\psi_{1,1}(z)=\frac{\sqrt{3}}{2}L_1(2z-1)=\frac{\sqrt{3}}{2}(2z-1),$$
$$\psi_{1,2}(z)=\frac{\sqrt{5}}{6}L_2(2z-1)=\frac{\sqrt{5}}{6}(6z^2-6z+1),$$
$$\psi_{1,3}(z)=\frac{\sqrt{7}}{20}L_3(2z-1)=\frac{\sqrt{7}}{20}(20z^3-30z^2+12z-1),$$
$$\psi_{1,4}(z)=\frac{\sqrt{9}}{70}L_4(2z-1)=\frac{3}{70}(70z^4-140z^3+90z^2-20z+1),$$
$$\psi_{1,5}(z)=\frac{\sqrt{11}}{252}L_5(2z-1)=\frac{\sqrt{11}}{252}(252z^5-630z^4+560z^3-210z^2+30z-1).$$

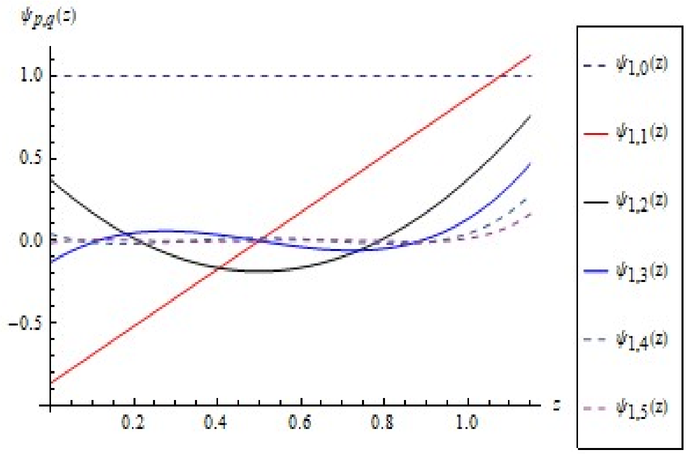

Figure 2: six OTWs for $p=1$, and $d=1$.

3. FUNCTION APPROXIMATION

A function $v(z)\in L^2(R)$ defined on $[0,1]$ can be express as linear combination of orthogonal Taylor wavelets (OTWs) series as

$$v(z)\cong\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}b_{p,q}\psi_{p,q}(z), \tag{5}$$

where ,

$b_{p,q}=\langle v(z),\psi_{p,q}(z)\rangle$ and symbol $\langle .,. \rangle$ indicate the inner product. If the given infinite series in above equation (5) is truncated, it could be written as:

$$v(z)\cong\sum_{p=1}^{2^{d-1}}\sum_{q=0}^{Q-1}b_{p,q}\psi_{p,q}(z)=B^T\psi(z), \tag{6}$$

where,

B and $\psi(z)$ are $2^{d-1}Q\times 1$ matrices defined by as

$$B=[b_{1,0},\dots,b_{1,Q-1},b_{2,0},\dots,b_{2,Q-1},\dots,b_{2^{d}-1,0},\dots,b_{2^{d}-1,Q-1}]^T, \tag{7}$$

$$\psi(z)=[\psi_{1,0}(z),\dots,\psi_{1,Q-1}(z),\psi_{2,0}(z),\dots,\psi_{2,Q-1}(z),\dots,\psi_{2^{d-1},0}(z),\dots,\psi_{2^{d-1},Q-1}(z)]^T. \tag{8}$$

3.1 CONVERGENCE OF ORTHOGONAL TAYLOR WAVELETS

Theorem: If any continuous function $v(z)\in L^2(R)$ be bounded and described on [0,1) , such that $v(z)\le d$, then the OTWs series expansion of $v(z)$ uniformly convergent [26].

Proof: Let the continuous function $v(z)$ be bounded function on [0,1) then OTWs coefficients of $v(z)$ is defined by

$$b_{p,q}=\int_0^1 v(z)\psi_{p,q}(z)dz,$$
$$=\int_{\frac{p-1}{2^{d-1}}}^{\frac{p}{2^{d-1}}}v(z)\sqrt{2^{d-1}}U_q(2^{d-1}z-p+1)dz. \tag{9}$$

Now, putting $2^{d-1}z-p+1=n$ in eq. (9) we get,

$$b_{p,q}=\int_{\frac{p-1}{2^{d-1}}}^{\frac{p}{2^{d-1}}}v\left(\frac{n-1+p}{2^{d-1}}\right)\sqrt{2^{d-1}}U_q(n)2^{-d+1}dn$$
$$=\sqrt{2^{-d+1}}\int_0^1 v\left(\frac{n-1+p}{2^{d-1}}\right)U_q(n)dn. \tag{10}$$

By applying generalized mean value (GMV) theorem,

$$=\sqrt{2^{-d+1}}\,v\left(\frac{x-1+p}{2^{d-1}}\right)\int_0^1 U_q(n)\,dn,$$

for any $x\in(0,1)$

$$=\sqrt{2^{-d+1}}\,v\left(\frac{x-1+p}{2^{d-1}}\right)\mathfrak{I},$$

where,

$$\mathfrak{I}=\int_0^1 U_q(n)dn,$$

then,

$$|b_{p,q}|=\left|\sqrt{2^{-d+1}}\right|\left|v\left(\frac{x-1+p}{2^{d-1}}\right)\mathfrak{I}\right|.$$

Since continuous function $v(z)$ is bounded, then the series $\sum_{p,q=0}^{\infty}b_{p,q}$ is absolutely convergent. Hence the OTWs series of continuous function $v(z)$ is uniformly convergent.

4. PROCEDURE OF SOLUTION

Consider one- dimensional PDEs is of the form,

$$\frac{\partial^2 v}{\partial z^2}+\delta\frac{\partial v}{\partial z}+\beta v=g(z), \tag{11}$$

with the conditions

$$v(0)=a;\quad v(1)=b, \tag{12}$$

Where δ, β are either constant or a function of z or function of v and the function $g(z)$ is continuous.

The above equation (11) is now written as

$$\Re(z)=\frac{\partial^2 v}{\partial z^2}+\delta\frac{\partial v}{\partial z}+\beta v-g(z)\ , \tag{13}$$

where,
$\Re(z)$ is the residual for equation (11). When $\Re(z)=0$, for the exact solution, $v(z)$ only which satisfy the given boundary conditions. Let test series solution of given One- dimensional PDE (11), a function $v(z)$ defined over $[0,1)$ may be express approximately by using orthogonal Taylor wavelets (OTWs) series with weight function $w(z)=z(1-z)$ as

$$v(z)=\sum_{p=1}^{2^{d-1}}\sum_{q=0}^{Q-1}b_{p,q}\,\Psi_{p,q}(z), \tag{14}$$

where $\Psi_{p,q}(z)=w(z)\times\psi_{p,q}(z)$ and $b_{p,q}{'s}$ are unknown parameters to be find out. If we take higher degree orthogonal Taylor wavelets polynomials then solution accuracy definitely increased. Differentiating equation (14) twice with respect to z and putting the values of $\frac{\partial^2 v}{\partial z^2},\frac{\partial v}{\partial z},v$ in eq. (13). To find $b_{p,q}{'s}$ we chose weight function as supposed bases elements and integrating on boundary values together with the residual to zero [15]
i.e.,

$$\int_0^1\Psi_{1,q}(z)\Re(z)dz=0,\ q=0,1,2,\dots . \tag{15}$$

Thus we obtain system of linear equations with unknown parameters, on solving obtained system we get unknowns. Then substitute these unknowns' parameters in test solution, approximate solution of eq. (11) is received.

Algorithm:
Input: d,p,q,Q
Step1: Define OTWs $\psi_{p,q}(z)$ by using equations (3) and (4).
Step 2: define unknowns OTWs coefficients $b_{p,q}{'s}$.
Step 3: Consider one- dimensional PDE is of the form

$\frac{\partial^2 v}{\partial z^2}+\delta\frac{\partial v}{\partial z}+\beta v=g(z)$, using equation (11).

Step 4: Taking Residuals $\Re(z)=\frac{\partial^2 v}{\partial z^2}+\delta\frac{\partial v}{\partial z}+\beta v-g(z)$ by using step 3.
Step 5: Choose weight function $w(z)=z(1-z)$ as supposed basis element to satisfy boundary conditions given in equation (12).
Step 6: Take new OTWs basis as $\Psi_{p,q}(z)=w(z)\times\psi_{p,q}(z)$.
Step 7: Approximate $v(z)$ by using equation (14).
Step 8: Using step 7 calculate $\frac{\partial^2 v}{\partial z^2},\frac{\partial v}{\partial z},v$.
Step 9: Take algebraic system $\int_0^1\Psi_{1,q}(z)\Re(z)dz=0, q=0,1,2,\dots$ by using equation (15).
Step 10: Solve algebraic system given in step 9 to find unknowns coefficients $b_{p,q}{'s}$.

Output: $v(z)=\sum_{p=1}^{2^{d-1}}\sum_{q=0}^{Q-1}b_{p,q}\,\Psi_{p,q}(z)$.

5. NUMERICAL PROBLEMS

Problem 5.1 Consider one - dimensional PDE as [27, 28]

$$\frac{\partial^2 v}{\partial z^2}-\pi^2 v=-2\pi^2\sin(\pi z),\quad 0\le z\le 1, \tag{16}$$

with conditions:

$$v(0)=0;\quad v(1)=0\ . \tag{17}$$

Taken problem 5.1 has exact solution:

$v(z)=\sin(\pi z)$.

The executions of equation (16) as per the proposed OTWGNM of solution described in above section 4 are as follows. The "residual" of equation (16) can be written as:

$$\Re(z)=\frac{\partial^2 v}{\partial z^2}-\pi^2 v+2\pi^2\sin(\pi z)\ . \tag{18}$$

Now selecting the weight function $w(z)=z(1-z)$ for OTWs bases to satisfying the given conditions (17), i.e.,

$$\Psi(z)=w(z)\times\psi(z)$$
$$\Psi_{1,0}(z)=\psi_{1,0}(z)\times w(z)=z(1-z),$$
$$\Psi_{1,1}(z)=\psi_{1,1}(z)\times w(z)=\frac{\sqrt{3}}{2}(2z-1)z(1-z),$$
$$\Psi_{1,2}(z)=\psi_{1,2}(z)\times w(z)=\frac{\sqrt{5}}{6}(6z^2-6z+1)z(1-z).$$

Let the solution of (16) for $d=1$ and $q=3$ is given by

$$v(z)=b_{1,0}\,\Psi_{1,0}(z)+b_{1,1}\,\Psi_{1,1}(z)+b_{1,2}\Psi_{1,2}(z), \tag{19}$$

Then the eq. (19) becomes,

$$v(z)=b_{1,0}z(1-z)+b_{1,1}\frac{\sqrt{3}}{2}(2z-1)z(1-z)+b_{1,2}\frac{\sqrt{5}}{6}(6z^2-6z+1)z(1-z), \tag{20}$$

Differentiating eq. (20) twice w .r. t. z we get,

$$\frac{\partial v}{\partial z}=b_{1,0}(1-2z)+b_{1,1}(-3\sqrt{3}z^2+3\sqrt{3}z-\frac{\sqrt{3}}{2})+b_{1,2}(-4\sqrt{5}z+6\sqrt{5}z^2-\frac{7\sqrt{5}}{3}z+\frac{\sqrt{5}}{6}), \tag{21}$$

$$\frac{\partial^2 v}{\partial z^2} = b_{1,0}(-2) + b_{1,1}(-6\sqrt{3}\,z + 3\sqrt{3}) + b_{1,2}(-12\sqrt{5}\,z^2 + 12\sqrt{5}\,z - \frac{7\sqrt{5}}{3}) \quad (22)$$

Using eq. (20) and (22), then eq. (18) becomes,

$$\begin{aligned}\Re(z) &= b_{1,0}(-2) + b_{1,1}(-6\sqrt{3}\,z + 3\sqrt{3}) \\ &+ b_{1,2}(-12\sqrt{5}\,z^2 + 12\sqrt{5}\,z - \frac{7\sqrt{5}}{3}) \\ &- \pi^2[(b_{1,0}\,z(1-z) + b_{1,1}\sqrt{3}\,z(z-\frac{1}{2})\,(1-z) \\ &+ \sqrt{5}\,b_{1,2}\,z\,(z^2 - z + \frac{1}{6})(1-z)] \\ &+ 2\pi^2 \sin(\pi z) \\ &= b_{1,0}(-2 - \pi^2 z + \pi^2 z^2) \\ &+ b_{1,1}[3\sqrt{3} + z(\frac{\sqrt{3}}{2}\pi^2 - 6\sqrt{3}) \\ &- \frac{3\sqrt{3}}{2}\pi^2 z^2 + \sqrt{3}\pi^2 z^3] \\ &+ b_{1,2}[-\frac{7\sqrt{5}}{3} + (12\sqrt{5} - \frac{\sqrt{5}}{6}\pi^2)z \\ &+ (-12\sqrt{5} + \frac{7\sqrt{5}}{6}\pi^2)z^2 - 2\sqrt{5}\pi^2 z^3 \\ &+ \sqrt{5}\pi^2 z^4] \\ &+ 2\pi^2 \sin(\pi z).\end{aligned} \quad (23)$$

This $\Re(z)$ is the "residual" of equation (16). The "weight functions" are the same as the "bases functions". So, by the weighted Galerkin method (WGM), we consider the following:

$$\int_0^1 \Psi_{1,q}(z)\Re(z)dz = 0, q = 0,1,2. \quad (24)$$

For $q = 0,1,2$ in eq. (24),

i.e.,

$$\int_0^1 \Psi_{1,0}(z)\Re(z)dz = 0, \quad \int_0^1 \Psi_{1,1}(z)\Re(z)dz = 0, \quad \int_0^1 \Psi_{1,2}(z)\Re(z)dz = 0 ,$$

then this implies that

$$-(0.66232)b_{1,0} + (0)b_{1,1} + (0.0598755)b_{1,2} + 2.54648 = 0, \quad (25)$$

$$(0)b_{1,0} - (0.185249)b_{1,1} + (0)b_{1,2} + 0.00 = 0, \quad (26)$$

$$(0.0598755)b_{1,0} + (0)b_{1,1} - (0.0369508)b_{1,2} - 0.280079 = 0. \quad (27)$$

We have three equations (25)-(27) with three unknown coefficients, i.e., $b_{1,0}, b_{1,1}, b_{1,2}$. by solving this system of algebraic equations; we obtain the values of coefficients:

$b_{1,0} = 3.7018339170848553$, $b_{1,1} = 0.0000000000000000$,
$b_{1,2} = -1.5812788749879645$.

So, by substituting the values of coefficients $b_{1,0}, b_{1,1}, b_{1,2}$. in equation (20) we get the OTWGNM (approx.) solution of problem 5.1. Comparison between OTWGNM (approx.), exact and other available methods solution are given in table-1. Absolute error $= |v_a(z) - v_e(z)|$, (here $v_a(z)$ stand for approx. solution and $v_e(z)$ stand for exact solution) given below in table-2 and comparison of OTWGNM (approx.) and exact solution of problem 5.1 represented in figure-3.

Table-1: Comparison between OTWGNM (approx.), exact and other available methods solution for problem 5.1

z	Numerical solution				Exact solution
	FDM in Ref [28]	Ref [29]	HWGM in Ref [28]	OTWGNM	
0.1	0.310289	0.308865	0.308754	0.308768	0.309016
0.2	0.590204	0.587527	0.588509	0.588522	0.588772
0.3	0.812347	0.808736	0.809554	0.809561	0.809016
0.4	0.954971	0.950859	0.950670	0.950671	0.951056
0.5	1.004126	0.999996	0.999123	0.999122	1.000000
0.6	0.954971	0.951351	0.950670	0.950671	0.951056
0.7	0.812347	0.809671	0.809554	0.809561	0.809016
0.8	0.590204	0.588815	0.588509	0.588522	0.587785
0.9	0.310289	0.310379	0.308754	0.308768	0.309016

Table-2: Absolute errors comparison for problem 5.1.

z	Absolute errors			
	FDM in Ref [28]	Ref [29]	HWGM in Ref [28]	OTWGNM
0.1	1.27E-03	1.51E-04	2.60E-04	2.48E-04
0.2	1.43E-03	1.25E-03	2.60E-04	2.50E-04
0.3	3.33E-03	2.80E-04	5.40E-04	5.45E-04
0.4	3.92E-03	1.97E-04	3.90E-04	3.85E-04
0.5	4.13E-03	4.00E-06	8.80E-04	8.78E-04
0.6	3.92E-03	2.95E-04	3.90E-04	3.85E-04
0.7	3.33E-03	6.55E-04	5.40E-04	5.45E-04
0.8	2.42E-03	1.03E-03	7.20E-04	7.37E-04
0.9	1.27E-03	1.36E-03	2.60E-04	2.38E-04

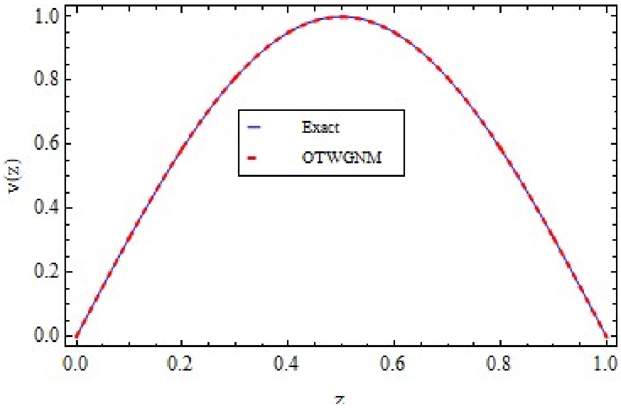

Figure 3: Comparison between OTWGNM (approx.) and exact solution for problem 5.1.

Problem 5.2 Take one- dimensional [13, 28]

$$\frac{\partial^2 v}{\partial z^2}-\frac{\partial v}{\partial z}=-(e^{z-1}+1),\ 0\le z\le 1, \tag{28}$$

with conditions

$$v(0)=0, \& \, v(1)=0. \tag{29}$$

And exact solution to problem 5.2 is as follows:

$v(z)=z\,(1-e^{z-1})$.

After employing the method, which is mention in section 4, we get these constants

$b_{1,0}=0.7966101110338866,\ b_{1,1}=0.2100496804 7284323,$

$b_{1,2}=0.0521358446 7873635.$

So, by substituting above coefficients values in equation (20), we get the OTWGNM (approx.) solution of second test problem 5.2. Obtained OTWGNM solutions are comparing with exact and other established method solutions are presented in table-3 and there absolute errors are given in table-4. Comparison of exact and OTWGNM solution of problem 5.2 represented in figure-4.

Table 3: Comparison of the exact and OTWGNM solution for problem 5.2

z	Numerical solution			Exact solution
	FDM in Ref [28]	Ref [27]	OTWGNM	
0.1	0.061948	0.059383	0.059401	0.059343
0.2	0.115151	0.110234	0.110119	0.110134
0.3	0.158162	0.151200	0.150947	0.151102
0.4	0.189323	0.180617	0.180403	0.180475
0.5	0.206737	0.196983	0.196724	0.196735
0.6	0.208235	0.198083	0.197866	0.197808
0.7	0.191342	0.181655	0.181508	0.181427
0.8	0.153228	0.145200	0.145045	0.145015
0.9	0.090672	0.085710	0.085596	0.085646

Table 4: Comparison of absolute errors for discussed problem 5.2.

z	Absolute error		
	FDM in Ref [28]	Ref [27]	OTWGNM
0.1	2.61E-03	4.00E-05	5.88E-05
0.2	5.02E-03	1.00E-04	1.54E-05
0.3	7.14E-03	1.76E-04	7.74E-05
0.4	8.85E-03	1.42E-04	7.23E-05
0.5	1.00E-02	2.48E-04	1.08E-05
0.6	1.04E-02	2.75E-04	5.82E-05
0.7	9.92E-03	2.28E-04	8.03E-05
0.8	8.21E-03	1.85E-04	2.97E-05
0.9	5.03E-03	6.40E-05	4.96E-05

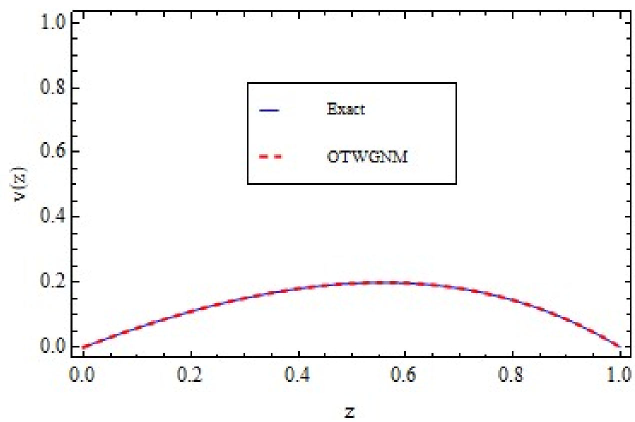

Figure 4: Comparison between OTWGNM (approx.) and exact solution for problem 5.2.

Problem 5.3 Take another one- dimensional PDE [28, 29]

$$\frac{\partial^2 v}{\partial z^2}+v=-z,\quad 0\le z\le 1, \tag{30}$$

with conditions,

$$v(0)=0, \& \, v(1)=0. \tag{31}$$

Exact solution to problem 5.3: $v(z)=\dfrac{\sin(z)}{\sin(1)}-z$.

By applying OTWGNM which is mention in section 4, we get the following unknown coefficients,

$b_{1,0}=0.2770345596432553,\ b_{1,1}=0.0985719971787166,$

$b_{1,2}=-0.010469883506687955.$

So, by substituting above obtained coefficients values into equation (20) we get the OTWGNM (approx.) solution of problem 5.3. Obtained OTWGNM solutions are compared with exact and other established method solutions are presented in table 2.5. Absolute errors comparison of proposed OTWGNM with other available methods FDM, HWGM is provided in table

2.6. And graph of comparison between exact and OTWGNM (approx.) solution to problem 5.3, is depicted in figure 2.5.

Table 2.5: Comparison between FDM, HWGM, exact and OTWGNM (approx.) solution for discussed problem 5.3

z	Numerical solution			Exact solution
	FDM in Ref [28]	HWGM in Ref [28]	OTWGNM	
0.1	0.018660	0.018624	0.0186252	0.0186415
0.2	0.036132	0.036102	0.0361054	0.0360977
0.3	0.051243	0.051214	0.0512196	0.0511948
0.4	0.062842	0.062793	0.0628028	0.0627829
0.5	0.069812	0.069734	0.0697464	0.0697469
0.6	0.071084	0.070983	0.0709979	0.0710184
0.7	0.065646	0.065545	0.0655610	0.0655851
0.8	0.052550	0.052481	0.0524957	0.0525025
0.9	0.030930	0.030908	0.0309179	0.0309019

Table 2.6: Absolute errors comparison of OTWGNM, FDM and HWGM for problem 5.3

z	Absolute error		
	FDM in Ref [28]	HWGM in Ref [28]	OTWGNM
0.1	1.80E-05	1.80E-05	1.63E-05
0.2	3.40E-05	4.00E-06	7.77E-06
0.3	4.80E-05	1.90E-05	2.48E-05
0.4	5.9005	1.00E-05	1.99E-05
0.5	6.50E-05	1.30E-05	5.86E-07
0.6	6.60E-05	3.50E-05	2.04E-05
0.7	6.10E-05	4.00E-05	2.41E-05
0.8	4.80E-05	2.10E-05	6.78E-06
0.9	2.80E-05	6.00E-06	1.60E-05

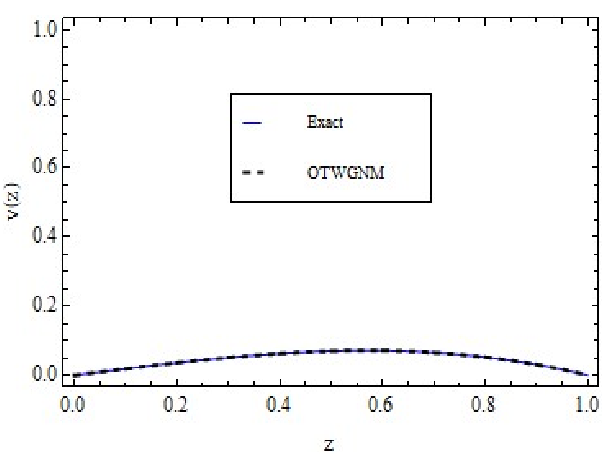

Figure 2.5: Comparison between OTWGNM (approx.) and exact solution for problem 5.3.

Problem 5.4 Finally, consider another one- dimensional PDE [28, 30]

$$\frac{\partial^2 v}{\partial z^2}+\frac{8}{z}\frac{\partial v}{\partial z}+zv=z^5-z^4+44z^2-30z \qquad ,0\le z\le 1, \quad (32)$$

with conditions:

$$v(0)=0\,,\&\ v(1)=0. \qquad (33)$$

Exact solution to problem 5.4 is $v(z)=-z^3+z^4$.

By applying OTWGNM which is mention in section 4, we get the value of unknown coefficients

$b_{1,0}=-0.3333333333333326$, $b_{1,1}=-0.5773502691896257$, $b_{1,2}=-0.447213595499957$.

So, by substituting above coefficients values in equation (20) we obtain the OTWGNM (approx.) solution of problem 5.4. Obtained OTWGNM (approx.) solutions are compared with exact and other established methods (FDM, HWGM) solutions are given in table 2.7. Absolute errors comparison are provided in table 2.8, and figure 2.6 shows a comparison of the proposed OTWGNM (approx.) and the exact solution for problem 5.4.

Table 2.7: Comparison of OTWGNM (approx.) and exact solution for problem 5.4.

z	Approximate solution			Exact solution
	FDM in Ref [28]	HWGM in Ref [28]	OTWGNM	
0.1	0.024647	-0.000900	-0.000900	-0.000900
0.2	0.024538	-0.006401	-0.006400	-0.006400
0.3	0.016024	-0.018904	-0.018900	-0.018900
0.4	-0.00072	-0.38407	-0.038400	-0.038400
0.5	-0.022021	-0.062512	-0.062500	-0.062500
0.6	-0.045926	-0.086417	-0.086400	-0.086400
0.7	-0.065532	-0.102920	-0.102900	-0.102900
0.8	-0.072190	-0.102420	-0.102400	-0.102400
0.9	-0.054840	-0.072914	-0.072900	-0.072900

Table 2.8: Absolute errors comparison for discussed problem 5.4

z	Absolute error		
	FDM in Ref [28]	HWGM in Ref [28]	OTWGNM
0.1	2.55E-02	0	2.29E-17
0.2	3.09E-02	1.00E-06	1.90E-17
0.3	3.40E-02	4.00E-06	1.38E-17
0.4	3.83E-02	7.00E-06	6.93E-18
0.5	4.05E-02	1.20E-05	2.77E-17
0.6	4.05E-02	1.70E-05	2.77E-17
0.7	3.74E-02	2.00E-05	2.77E-17
0.8	3.02E-02	2.00E-05	2.77E-17

0.9	1.81E-02	1.40E-05	0

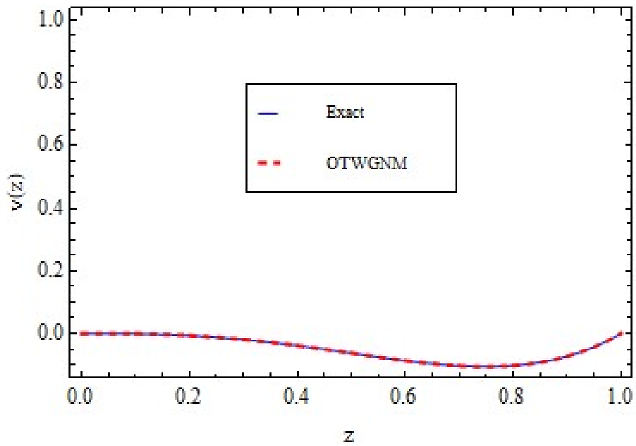

Figure 2.6: Comparison between OTWGNM (approx.) and exact solution for problem 5.4.

SIGNIFACENCE OF THE PROPOSED WORK

There are a lot of models in electrical engineering which are based on the one-dimensional PDEs (or ordinary differential equation) such as RLC circuit[19, 31, 32] RCC circuit [19], LC circuit [33], RC circuit [19] etc. All these circuits are basically used in electronics and electrical engineering. And these PDEs are used to as the mathematical model of electrical circuit and such kind of PDEs does not have exact numerical solutions. Therefore, there are several numerical methods are available to extract the numerical solutions of these mathematical models. But due to occur the more numerical errors in them, these are decreases the performance of the electrical devices. Keeping this type problem in the mind, we are presenting to the proposed method and get more accurate numerical solutions which may be helpful in the removing of such type difficulties for the electrical circuits. It is the most significant work (or contributions) of the proposed OTWGNM for the results of one- dimensional PDEs. And proposed work is compares with some existed wavelet Galerkin based methods; see in references [34-37].

CONCLUSION

In this paper, the certain one-dimensional PDEs have been solved by using OTWGNM. The derived results are compared to the results of other existed WGMs, FDM and exact solution, which demonstrate that the proposed OTWGNM is more credible and effective. Finally, tables and figures are used to provide a comparative study between exact and approximate solutions for discussed problems. For large value of Q we can get the results closure to exact solution. Hence proposed method is more effective for the results of mentioned problems and other kind of partial differential equation.

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