Volume 67, Issue 3, 2023
Journal of Scientific Research
of
The Banaras Hindu University

# Generalized Ratio Type Estimator under Adaptive Cluster Sampling

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**Abstract: The Adaptive Cluster Sample (ACS) sampling design is employed for the evaluation of scarce, concentrated, vulnerable, and complicated population parameters. Generalized ratio type estimator under ACS has been proposed in this paper. The theoretical formulas of the suggested estimator's mean square error is generated up to first order. Using the simulated x- and y-values from Chutiman and Kumphon (2008), the proposed estimator's effectiveness is empirically evaluated with certain known ratio and ACS estimators. Furthermore, the empirical optimum values of p and q are determined. The results indicated that when there is a positive correlation between the study variable and auxiliary variable then the suggested estimate under ACS is a reliable alternative to other estimators.**

**Index Terms: ACS; Ratio Estimator; Bias; Mean Square Error; Efficiency.**

## I. Introduction

It is extremely complicated to get the population mean of uncommon animals, plants, hidden, or indigenous species, uneven mineral mining, pollution concentrations, noise disturbances, drug users, HIV infected patients, thieves, and hot spot inquiries. In these cases Adaptive Cluster Sampling is the effective option of sampling design with the use of ratio type estimators. Thompson suggested this sampling procedure in 1990. In cases, when the area is broad, heterogeneous, and responsive to change, the design is most robust. The estimators in ACS are classified into two categories: the first category includes modified Hansen and Hurwitz (1943) and Horvitz and Thompson (1952) estimators while the second category includes ratio-type estimators that take use of auxiliary data. In ratio method of estimation the auxiliary variable plays an important role to get the precise estimate of the study variable where the auxiliary variable is correlated with the study variable positively and satisfying the relation $\frac{1C_x}{2C_y} \leq \rho \leq 1$ . Several authors like Sharma and Bhatnagar (2008), Jeelani et al., (2017), Kumar et al., (2018) Hussain et al., (2021) have presented estimators for the estimation of population mean that yields better results than the usual sample mean. In ACS, The initial sample in ACS is collected using a probability sampling design.. A specific condition is defined in advance in order to adopt the neighbouring units. If the specified condition is satisfied in the initially selected samples, the neighbouring units are examined and added to the sample, this process continues until the neighbouring units don't satisfy the condition. The units that meet the condition form the network and the units that don't meet the condition are called edge units. The collection of network and edge units is called as cluster. The neighbourhood are the spatially adjacent units in the east, west, north and south of the selected units that satisfies the pre-specified condition. The figure 1 illustrates an example of a cluster having network with its associated edge units as four spatially adjacent units. The unit with a star (*) is an initially selected unit, the condition of adaption is pre-defined as a unit greater than or equal to 1 will be added to the network. The unit which are in the east, west, north and south of the initially selected sample are known as first order neighbourhood. The units in the shaded area form a network while the units in bold numbers are respective edge units of a network. The network with its associated edge units makes up a cluster.

| | | | | | |
|---|---|---|---|---|---|
| | | **0** | | | |
| | **0** | 8 | **0** | | |
| **0** | 3 | 4* | 10 | **0** | |
| | **0** | 8 | 6 | 2 | **0** |
| | | **0** | 7 | **0** | |
| | | | **0** | | |

Fig. 1. Cluster (Network and Edge Units)

## II. Manuscript Organization

The ratio estimator is defined as the division of means of two random variables. Several authors have used the ratio estimator in order to increase the precision of the estimators. The pioneer work has been done by Cochran (1940). The proposed estimator and the mean square error are given as $\hat{\hat{\bar{Y}}}_R = \bar{y}\left(\frac{\mu_x}{\bar{x}}\right)$ and $MSE\ (\hat{\hat{\bar{Y}}}_R) = \theta\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho_{xy}C_xC_y\right)$. With due course of time Sisodia and Dwivedi (1981) used coefficient of variation as an auxiliary variable and proposed ratio estimator as:

$$t_1 = \bar{y}\left(\frac{\mu_x + C_x}{\bar{x} + C_x}\right)$$

$$MSE(t_1) = \theta{\mu_y}^2\left(C_y^2 + \theta_1 C_x^2 - 2\theta_1\rho_{xy}C_xC_y\right) \quad (1)$$

$$\text{where } \theta_1 = \frac{\mu_x}{\mu_x + C_x}$$

Upadhyay and Singh (1999) made the use of auxiliary information as kurtosis and coefficient of variation and proposed the estimator as:

$$t_2 = \bar{y}\left(\frac{\beta_{2(x)}\mu_x + C_x}{\beta_{2(x)}\bar{x} + C_x}\right)$$

$$MSE(t_2) = \theta{\mu_y}^2\left(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2\rho_{xy}C_xC_y\right) \quad (2)$$

$$\text{where } \theta_2 = \frac{\beta_{2(x)}\mu_x}{\beta_{2(x)}\mu_x + C_x}$$

$$t_3 = \bar{y}\left(\frac{C_x\mu_x + \beta_{2(x)}}{C_x\bar{x} + \beta_{2(x)}}\right)$$

$$MSE(t_3) = \theta{\mu_y}^2\left(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3\rho_{xy}C_xC_y\right) \quad (3)$$

$$\text{where } \theta_3 = \frac{C_x\mu_x}{C_x\mu_x + \beta_{2(x)}}$$

Singh and Tailor (2003) suggested ratio type estimator with the use of correlation coefficient as:

$$t_4 = \bar{y}\left(\frac{\mu_x + \rho_{xy}}{\bar{x} + \rho_{xy}}\right)$$

$$MSE(t_4) = \theta{\mu_y}^2\left(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4\rho_{xy}C_xC_y\right) \quad (4)$$

$$\text{where } \theta_4 = \frac{\mu_x}{\mu_x + \rho_{xy}}$$

Kadilar and Cingi (2003) suggested an estimator as:

$$t_5 = \bar{y}\frac{\mu_x^2}{\bar{x}^2}$$

$$MSE(t_5) = \theta\mu_y^2\left(C_y^2 + 4C_x^2 - 4\rho_{xy}C_xC_y\right) \quad (5)$$

Yan and Tian (2010) proposed ratio type estimator by taking skewness and kurtosis into account as auxiliary information:

$$t_6 = \bar{y}\left(\frac{\beta_{2(x)}\mu_x + \beta_{1(x)}}{\beta_{2(x)}\bar{x} + \beta_{1(x)}}\right)$$

$$MSE(t_6) = \theta\mu_y^2\left(C_y^2 + \theta_6^2 C_x^2 - 2\theta_5\rho_{xy}C_xC_y\right) \quad (6)$$

$$\text{where } \theta_5 = \frac{\beta_{2(x)}\mu_x}{\beta_{2(x)}\mu_x + \beta_{1(x)}}$$

$$t_7 = \bar{y}\left(\frac{\beta_{1(x)}\mu_x + \beta_{2(x)}}{\beta_{1(x)}\bar{x} + \beta_{2(x)}}\right)$$

$$MSE(t_7) = \theta\mu_y^2\left(C_y^2 + \theta_7^2 C_x^2 - 2\theta_6\rho_{xy}C_xC_y\right) \quad (7)$$

$$\text{Where } \theta_6 = \frac{\beta_{1(x)}\mu_x}{\beta_{1(x)}\mu_x + \beta_{2(x)}}$$

The usual notations used in the above derivations are defined as follows:

$$\theta = \frac{N-n}{Nn} = \frac{1-f}{n}, f = \frac{n}{N}, C_y = \frac{S_y}{\mu_y}, C_x = \frac{S_x}{\mu_x}$$

$$s_y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i - \mu_y\right)^2$$

$$s_y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i - \mu_y\right)^2, \rho_{xy} = \frac{S_{xy}}{S_xS_y}$$

$$S_{xy} = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \mu_x)\left(y_i - \mu_y\right).$$

But, there are situations when the population is rare and a researcher can't get information about the population as a whole, but they can get information about the clusters. In handling such situations ACS is used. Thompson (1990) proposed the unbiased mean estimator based on the modification of Hansen-Hurwitz (HH) estimator as:

$$\hat{\mu}_y^R = t_8 = \frac{1}{n}\sum_{i=1}^{n} w_{yi}$$

$$\text{The variance of } \hat{\mu}_y^R = \theta\mu_{wy}^2 C_{wy}^2 \quad (8)$$

$$\text{where } \theta = \frac{N-n}{Nn} = \frac{1-f}{n}, f = \frac{n}{N} \text{ and } C_{wy}^2 = \frac{S_{wy}^2}{\mu_{wy}^2}$$

Later Chutiman (2013) proposed ratio type estimators on the basis of Dryver and Chao (2007) as:

$$t_9 = \bar{w}_y\left(\frac{\mu_x + C_{wx}}{\bar{w}_x + C_{wx}}\right)$$

$$t_{10} = \bar{w}_y\left(\frac{\mu_{x\beta_{2(wx)}} + C_{wx}}{\bar{w}_{x\beta_{2(wx)}} + C_{wx}}\right)$$

$$t_{11} = \bar{w}_y\left(\frac{\mu_x + \beta_{2(wx)}}{\bar{w}_x + \beta_{2(wx)}}\right)$$

The mean square error of $(t_9)$, $(t_{01})$, $(t_{12})$, is given by:

$$MSE(t_9) = \theta\mu_{wy}^2\left(C_{wy}^2 + \theta_{w7}^2 C_{wx}^2 - 2\theta_{w7}\rho_{wxwy}C_{wx}C_{wy}\right) \quad (9)$$

$$\text{Where } \theta_{w7} = \frac{\mu_{wx}}{\mu_{wx} + C_{wx}}$$

$$MSE(t_{10}) = \theta\mu_{wy}^2\left(C_{wy}^2 + \theta_{w8}^2 C_{wx}^2 - 2\theta_{w8}\rho_{wxwy}C_{wx}C_{wy}\right) \quad (10)$$

$$\text{Where } \theta_{w8} = \frac{\mu_x\beta_{2(wx)}}{\mu_x\beta_{2(wx)} + C_{wx}}$$

$$MSE(t_{11}) = \theta\mu_{wy}^2\left(C_{wy}^2 + \theta_{w9}^2 C_{wx}^2 - 2\theta_{w9}\rho_{wxwy}C_{wx}C_{wy}\right) \quad (11)$$

$$\text{Where } \theta_{w9} = \frac{\mu_x}{\mu_x + \beta_{2wx}}$$

Let the usual finite population consists N distinct units labelled from 1,2,…, N. the variables $y_i$ and $x_i$ (i=1,2,3…..,N) denote the ith value for the survey and auxiliary variables respectively, 'n' denote the initial sample size. Let the population is divided into K exhaustive networks where $\varphi_i$ denotes the network that includes i units with $m_i$ number of units in the ith network. The mean, standard deviation, coefficient of variation, correlation coefficient and covariance of survey and auxiliary variable at network level is denoted by $\mu_{wy}, \mu_{wx}, \sigma_{wy}, \sigma_{wx}, C_{wy}, C_{wx}, \rho_{wxwy}, \sigma_{wywx}$ respectively.

The parameters of study and auxiliary variables are as:

**Study variable**

$$\mu_y = N^{-1}\sum_{i=1}^{N} y_i$$

$$\sigma_y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i-\mu_y\right)^2$$

**Auxiliary variable**

$$\mu_x = N^{-1}\sum_{i=1}^{N} x_i$$

$$\sigma_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i-\mu_x)^2$$

Let $w_{yi} = \frac{1}{m_i}\sum_{j\epsilon\varphi_i} y_i$ and $w_{xi} = \frac{1}{m_i}\sum_{j\epsilon\varphi_i} x_i$ be the transformed survey and auxiliary variables in the ith network respectively.

The transformed population parameters of study and auxiliary variables are:

**Study variable**

$$\mu_{wy} = N^{-1}\sum_{i=1}^{N} w_{yi}$$

$$\sigma_{wy}^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(w_{yi}-\mu_{wy}\right)^2$$

**Auxiliary variable**

$$\mu_{wx} = N^{-1}\sum_{i=1}^{N} w_{xi}$$

$$\sigma_{wx}^2 = (N-1)^{-1}\sum_{i=1}^{N}(w_{xi}-\mu_{wx})^2$$

The transformed population statistics of study and auxiliary are:

**Study variable**

$$\bar{w}_y = n^{-1}\sum_{i=1}^{n} w_{yi}$$

$$s_{wy}^2 = (n-1)^{-1}\sum_{i=1}^{n}\left(w_{yi}-\bar{w}_y\right)^2$$

**Auxiliary variable**

$$\bar{w}_x = n^{-1}\sum_{i=1}^{n} w_{xi}$$

$$s_{wx}^2 = (n-1)^{-1}\sum_{i=1}^{n}(w_{xi}-\bar{w}_x)^2$$

## III Results and Discussion

The proposed the generalized class of ratio type estimator under ACS as:

$$\hat{\mu}_y^r = t'_{p,q} = \bar{w}_y\left(\frac{p\beta_{1(wx)}\mu_x+\beta_{2(wx)}}{q\beta_{1(wx)}\bar{w}_x+\beta_{2(wx)}}\right),$$

where p and q are the constants to be determined in order to improve the efficiency of the proposed estimator.

The Mean square error of $\hat{\mu}_y^r = t'_{p,q}$ is given as:

$$\mathrm{MSE}\left(\hat{\mu}_y^r\right) = t'_{p,q} = \mu_y^2\Big[(\theta_{w10}-1)^2+\theta_{w10}^2\theta C_{wy}^2 + \theta_{w10}\theta_{w11}^2\theta C_{wx}^2(3\theta_{w13}-2) + \theta_{w10}\theta_{w11}\theta\rho_{wxwy}C_{wx}C_{wy}(2-4\theta_{w10})\Big] \quad (12)$$

Where $\theta_{w10} = \frac{p\beta_{1(wx)}\mu_{wx}+\beta_{2(wx)}}{q\beta_{1(wx)}\mu_{wx}+\beta_{2(wx)}}$

$$\theta_{w11} = \frac{q\beta_{1(wx)}\mu_{wx}}{q\beta_{1(wx)}\mu_{wx}+\beta_{2(wx)}}$$

**Special cases:**

The proposed generalized estimator is an alternative to different types of estimators as already existing in literature, shown through different cases as follows;

**(i)** For p = q = 1 the proposed generalized estimator becomes equal to $\bar{w}_y\left(\frac{\beta_{1(wx)}\mu_x+\beta_{2(wx)}}{\beta_{1(wx)}\bar{w}_x+\beta_{2(wx)}}\right)$ which is same as the estimator developed by S. K. Yadav et al. (2016). So it is found that the estimator proposed by S. K. Yadav et al. (2016) is a special case of the proposed generalized estimator.

**(ii)** For $p = q = \beta_{1(wx)} = 1$ , and $\beta_{2(wx)} = 0$ the proposed generalized estimator reduces to $\bar{w}_y\left(\frac{\mu_x}{\bar{w}_x}\right)$, which is same as the estimator developed by Dryver and Chao, (2007). So it is found that the estimator proposed by Dryver and Chao, (2007) is a special case of the proposed generalized estimator.

## IV Numerical Study

In order to evaluate the performance of proposed estimator, the simulated data of x-values and y-values from Chutiman and Kumphon (2008) were studied. The data statistics is given in table I, the MSE values of proposed estimators for different values of p and q are given in table II, and percentage relative efficiency is given in table III.

Table I. Summary Statistics

| | | | |
|---|---|---|---|
| $\mu_{wy}$=1.2225 | $\mu_{wx}$=0.550 | $\bar{w}_y$=1.2225 | $\bar{w}_x$=0.550 |
| $\beta_{1(wx)}$=7.953 | $\beta_{2(wx)}$=91.369 | $C_{wx}$=3.510 | $C_{wy}$=2.914 |
| $S_{wx}$=1.948 | $S_{wy}$=3.562 | $S_{wxwy}$=6.428 | $\rho_{wxwy}$=0.926 |
| $\rho_{xy}$=0.910 | $C_x$=4.325 | $C_y$=4.131 | $\theta_1$=0.114 |
| $\theta_2$=0.042 | $\theta_3$=0.064 | $\theta_{w4}$=0.006 | $\theta_{w6}$=0.046 |
| $\beta_{1x}$=60832 | $\beta_{2x}$=55.090 | N=400 | n=20 |

Table II.MSE of proposed estimator $t'_{p,q}$at different values of p and q

| $t'_{p,q}$ | $\theta_{w10}$ | $\theta_{w11}$ | **MSE** |
|---|---|---|---|
| $t'_{1,1}$ | 1.0000 | 0.0461 | 0.543 |
| $t'_{1,2}$ | 0.9559 | 0.0881 | 0.456 |
| $t'_{1,3}$ | 0.9156 | 0.1266 | 0.396 |
| $t'_{1,4}$ | 0.8785 | 0.1619 | 0.355 |
| $t'_{1,5}$ | 0.8444 | 0.1946 | 0.329 |
| $t'_{1,6}$ | 0.8127 | 0.2247 | 0.313 |
| $t'_{1,7}$ | 0.7834 | 0.2527 | 0.304 |
| $t'_{2,6}$ | 1.0000 | 0.0881 | 0.491 |
| $t'_{2,8}$ | 0.9578 | 0.1266 | 0.418 |
| $t'_{2,7}$ | 0.9190 | 0.1619 | 0.367 |
| $t'_{2,1}$ | 0.7909 | 0.2787 | 0.290 |
| $t'_{3,2}$ | 1.0000 | 0.1266 | 0.447 |
| $t'_{3,3}$ | 0.9595 | 0.1619 | 0.385 |
| $t'_{3,4}$ | 0.9222 | 0.1946 | 0.341 |
| $t'_{3,8}$ | 0.8258 | 0.2787 | 0.282 |
| $t'_{4,3}$ | 1.0422 | 0.1266 | 0.481 |
| $t'_{4,4}$ | 1.0000 | 0.1619 | 0.408 |
| $t'_{4,6}$ | 0.9251 | 0.2247 | 0.318 |
| $t'_{5,4}$ | 1.0405 | 0.1619 | 0.437 |
| $t'_{5,10}$ | 0.8371 | 0.3257 | 0.255 |

From table II it has been found that the MSE of the proposed estimator shows irregular trend on increasing the values of $p$ and $q$. Among these the MSE is least at $p = 5\, and\, q = 10$. Therefore the proposed estimator at $p = 5, q = 10$ gives the optimum value of MSE and is therefore considered most efficient than Sisodia and Dwivedi (1981) Upadhyay and Singh (1999), Singh and Tailor (2003), Kadilar and Cingi (2003), Yan and Tian (2010), Thompson (1990), Chutiman (2013).

Table III. Percentage relative efficiency (PRE) of the proposed estimator with respect to existing estimators (EE), where the MSE of proposed estimator (PE) = 0.255.

| **Existing Estimator** | **MSE** | **PRE** |
|---|---|---|
| $t_1 = \bar{y}\left(\frac{\mu_x + C_x}{\bar{x} + C_x}\right)$ | 0.966 | **378.823** |
| $t_2 = \bar{y}\left(\frac{\beta_{2(x)}\mu_x + C_x}{\beta_{2(x)}\bar{x} + C_x}\right)$ | 0.407 | **159.607** |
| $t_3 = \bar{y}\left(\frac{C_x\mu_x + \beta_{2(x)}}{C_x\bar{x} + \beta_{2(x)}}\right)$ | 1.117 | **438.039** |
| $t_4 = \bar{y}\left(\frac{\mu_x + \rho_{xy}}{\bar{x} + \rho_{xy}}\right)$ | 0.527 | **206.666** |
| $t_5 = \bar{y}\frac{\mu_x^2}{\bar{x}^2}$ | 1.904 | **746.666** |
| $t_6 = \bar{y}\left(\frac{\beta_{2(x)}\mu_x + \beta_{1(x)}}{\beta_{2(x)}\bar{x} + \beta_{1(x)}}\right)$ | 0.311 | **121.960** |

| | | |
|---|---|---|
| $t_7 = \bar{y}\left(\frac{\beta_{1(x)}\mu_x + \beta_{2(x)}}{\beta_{1(x)}\bar{x} + \beta_{2(x)}}\right)$ | 1.068 | **418.823** |
| $t_8 = \frac{1}{n}\sum_{i=1}^{n} w_{yi}$ | 0.603 | **236.470** |
| $t_9 = \bar{w}_y\left(\frac{\mu_x + C_{wx}}{\bar{w}_x + C_{wx}}\right)$ | 0.432 | **169.411** |
| $t_{10} = \bar{w}_y\left(\frac{\mu_x\beta_{2(wx)} + C_{wx}}{\bar{w}_x\beta_{2(wx)} + C_{wx}}\right)$ | 0.309 | **121.176** |
| $t_{11} = \bar{w}_y\left(\frac{\mu_x + \beta_{2(wx)}}{\bar{w}_x + \beta_{2(wx)}}\right)$ | 0.595 | **233.333** |

From the above table it is found that at the optimum value (0.225) reveals that the proposed estimator is most efficient than existing estimators proposed by Sisodia and Dwivedi (1981) Upadhyay and Singh (1999), Singh and Tailor (2003), Kadilar and Cingi (2003), Yan and Tian (2010), Thompson (1990), Chutiman (2013). The percentage relative efficiency is shown with the help of bar diagram shown below:

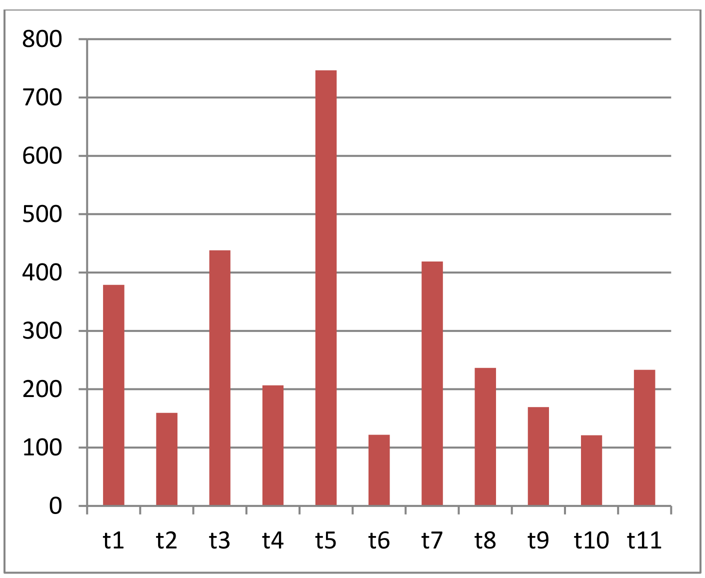


Fig. 2. PRE of various ratio type estimators w.r.t $t_{p,q}^{r2}$

The figure given above gives us the bird's eye view about the efficiency comparison of the proposed estimator while considering the optimum value of the proposed estimator. It reveals that the proposed estimator is most efficient than $t_5$ and less efficient than $t_6, t_{10}$ in comparison to other existing estimators taken into literature. Thus the proposed estimator $t_{5.10}^{r'}$ is best to obtain the precise estimate of the population mean in rare and clustered population when there is positive correlation between the variables at the unit level.

## V CONCLUSION

It is concluded that the generalized class of proposed estimator at different values of $p$ and $q$ performs better than the existing estimators compared on the basis of MSE. Among the different cases developed on the base of proposed estimator the estimator $t'_{5,10}$ is best and should be preferred for the estimation of population mean when population is rare and clustered.

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